NASA-CR- 171255

UDR-TR-84-87

TURBINE BLADE DAMPING STUDY

Robert J. Dominic

UNIVERSITY OF DAYTON RESEARCH INSTITUTE
300 College Park Avenue
Dayton, Ohio 45469

Performed for Marshall Space Flight Center
of NASA - Contract NAS 8-34682

November, 1984

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS
(Continued)

LIST OF TABLES

UNIVERSITY OF DAYTON
RESEARCH INSTITUTE
DAYTON, OHIO 45469

1. INTRODUCTION

This report describes the work performed by University of Dayton Research Institute (UDRI) for Marshall Space Flight Center (MSFC) of NASA under contract NAS8-34682, titled "Turbine Blade Damping Study." The work was directed to the analytical and experimental definition of the performance parameters of turbine blade platform friction dampers. Mr. Larry Kiefling of the Structural Dynamics Division of MSFC was the government's technical monitor for the work. The work was performed by the Vibration Analysis and Control Group, Michael Drake - Group Leader, of the Aerospace Mechanics Division, Dale H. Whitford - Supervisor, of UDRI. Robert Dominic was the project engineer for the work effort.

The contract was issued in early December of 1981, and the technical work effort was completed in June of 1984, with the reduction of data from the last experimental test series. The effort involved analytical studies utilizing the UDRI VAX 11/780 digital computer system and experimental studies in the UDRI laboratories and in a high-speed spin pit utilized under a subcontracted effort. The technical effort was based on a study of the first turbine stage of the high pressure fuel turbopump (HPFTP) of the space shuttle main engine (SSME), which has experienced blade fatigue problems.

2. BACKGROUND

Three catastrophic failures of HPFTP first stage turbine blades occurred during test stand runs early in the SSME development program. These failures were attributed to lockup of the platforms of adjacent blades, in one case due to welding of the

underplatform friction dampers to the platforms because of over-temperature conditions during the run, in another case due to extrusion of a nickelplate antifriction coating on the dampers into the interplatform gap, and in a third case due to an out of tolerance build that reduced or eliminated the interplatform gap for some blades in the stage. The mechanism of the failures was determined to be high cycle fatigue caused by excessive vibration of the blades. Failures occurred near the base of the airfoil section of the blades, just above the platform. Figure 1 shows two of the subject blades and the friction dampers that are placed in slots below the platforms. The dampers act on the bottom surface of the platforms, reducing the flexural vibrations of the blades through the dissipation of mechanical energy by friction heating. During pump operation the dampers also act to limit the leakage of cooling hydrogen, which is routed over the blade roots, into the turbine drive fluid stream. The dampers are forced against the under surface of the platforms by a combination of centrifugal force and the differential pressure between the cooling hydrogen and the turbine drive fluid.

The 63 blade turbine wheel is fed by 41 first stage nozzles. Thirteen shaft front bearing support struts are aligned with 13 of the nozzles in a necessarily unsymmetrical arrangement with 11 struts aligned three nozzles apart and two struts aligned four nozzles apart. Pressure pulses caused by the wakes off the nozzles, and particularly the higher amplitude pulses for the nozzles aligned with struts, excite vibrations in the blades that cause the high cycle fatigue problems. The repetition frequencies for these pulses are 10-1/4 per rev for struts spaced four nozzles apart, 13-2/3 per rev for struts spaced three nozzles apart, and 41 per rev for the symmetrically spaced nozzles. Sum and difference frequencies of these excitation components and their harmonics occur also to provide wide band excitation of the blades. The 13-2/3 per rev (14E) excitation of the blades is shown later to be a critical excitation frequency in the operating regime of the blades.

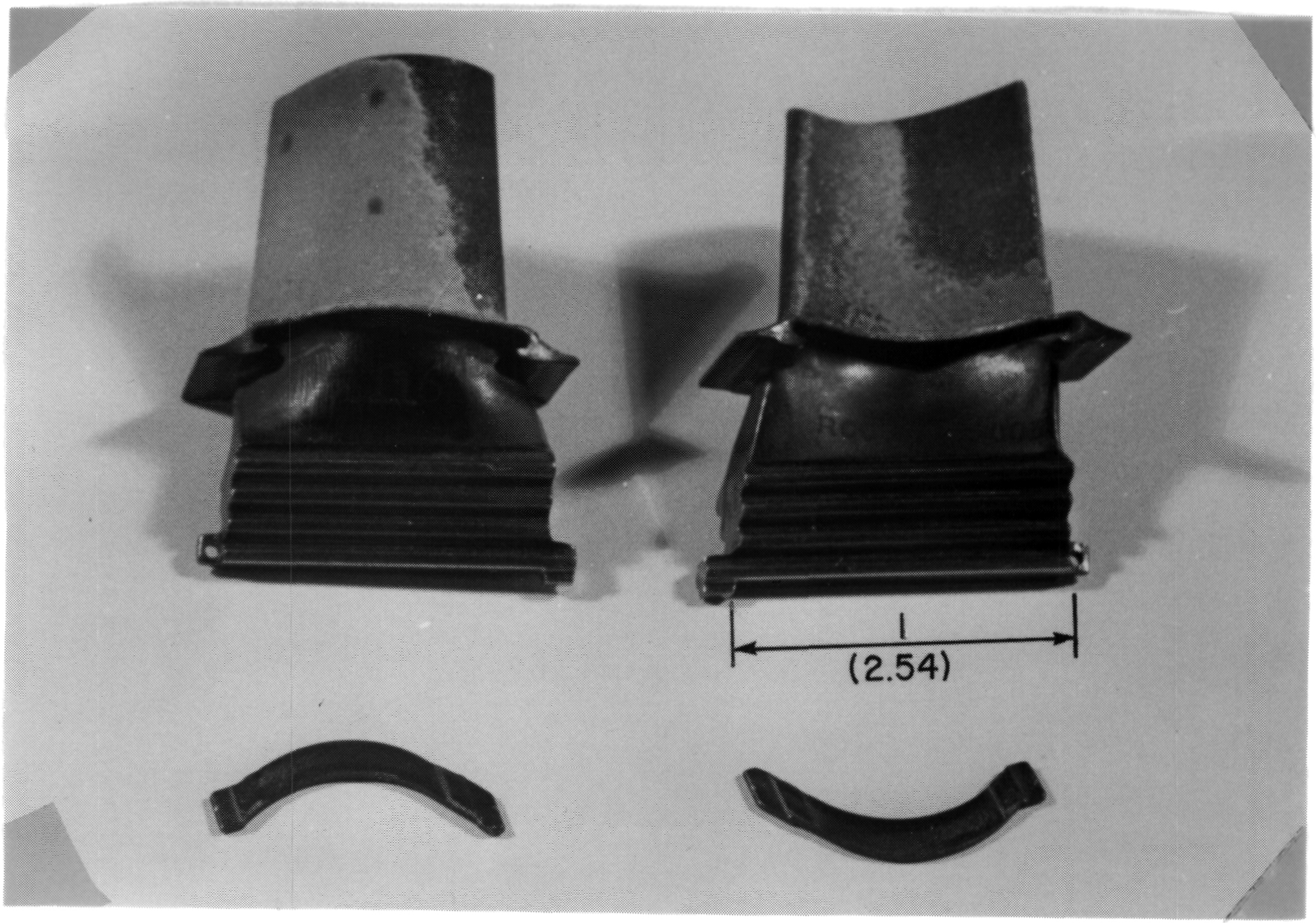


Figure 1 HPFTP Turbine Blades and Platform Friction Dampers

The nature of the blade fatigue failures caused them to be attributed to flexural resonance modes of the blades. Modal studies of the blades showed that the first two bending modes of the blade occurred at approximately 4,500 and 18,000 Hz. Later, during a whirligig spin test program conducted by Rocketdyne (Ref. 1), a resonance condition near 8,500 Hz was found. This resonance condition was first ascribed to the first torsional mode of the blade (which actually occurs at approximately 11,000 Hz), but it was later identified by UDRI as the first flexural mode of the airfoil section of the blade when the platforms are constrained from motion. The 14E excitation pulses occur at this airfoil-alone flexural resonance frequency of the blades during the long (relatively) time periods of engine operation at RPL.

As a result of the early studies the strut contour was changed to reduce the energy in the excitation pulses and the platform friction damper weight was reduced to provide more optimum damping. However, fatigue cracking continued to occur near the airfoil root with the platform at much lower than the specified and predicted life for the blades. Subsequently, UDRI contracted with NASA to evaluate the operation of the blade-damper system analytically and to evaluate the operation of a test system in a high speed spin pit.

3. OBJECTIVES

The objectives of this study, as stated in the contract, were:

1. to determine the structural damping inherent in typical blade-firtree installations of rocket propulsion engine turbopumps;
2. to identify the significant parameters affecting friction damping and dampers; and
3. to develop and evaluate improved friction damping mechanisms.

These objectives were to be accomplished through the testing of a bladed disk test assembly simulating the first stage turbine wheel of the HPFTP of the SSME, with theoretical analysis being conducted for correlation and optimization. Dampers were to be designed to provide a range of damping values from light damping to lockup for test and correlation with pre-test analysis. Significant damping/damper parameters were to be identified and a test matrix to evaluate the optimum values for each parameter or combination of parameters was to be designed and implemented to the extent that was possible. Then, recommendations for continued testing were to be made. UDRI has fulfilled the program objectives to the extent of the funding that was available, as shown in this report.

4. PROGRAM EFFORTS

In order to fulfull the objectives of the program, UDRI began parallel experimental and analytical efforts to evaluate the HPFTP first stage turbine blade platform friction damper performance parameters.

The first major tasks in the experimental effort were to fabricate a bladed disk assembly approximating the operational turbine wheel and to establish a methodology and obtain a facility for the performance of high speed spin tests of the test specimen. The first task was accomplished by contracting with Tech Development, Inc. of Dayton, Ohio for the design and fabrication of a turbine disk that would accept test HPFTP blades, which would be furnished by NASA. Also, a subcontract was signed with Applied Sensors International (ASI) of Cincinnati, Ohio for the provision of high speed spin testing services in their spin facility, including the use of an ASI slipring assembly to route strain gage signals from the spinning test turbine wheel to a data recording system. UDRI began a study to provide magnetic excitation of the bladed disk assembly to induce the first flexural vibration mode of the test blades in the spinning assembly.

UDRI embarked immediately on the analytical study through the use of a computer program based on the lumped parameter method

of turbine blade vibration analysis, as developed by Jones and Muszynska (Ref.2).

At the start of the investigation the emphasis of the program was directed to the alleviation of the first flexural resonance mode of the blade as cantilevered from the firtree root, which had been shown to occur at approximately 4500 Hz. This mode required 7E to 8E excitation of the blades to induce resonance at operational turbine speeds. Gradually, through study of the test data of Reference 1 and through evaluation of data from the analytical studies, emphasis was shifted to the flexural vibration mode of the airfoil section of the blade as if it were cantilevered from the blade platform. The platform can be rigidly clamped by high friction forces exerted by the dampers. This mode occurred at approximately 8500 Hz, as shown in the Reference 1 data, and required a 13E to 14E excitation forcing function at operational turbine speed.

Two other considerations evolved. First, for an airfoil-alone flexural mode with the platform fixed, root damping in the firtree could not act to control the resonance vibration nor would effective friction damping occur without platform motion. This was the situation for the early test stand turbine failures. Second, for the airfoil resonance mode the deflection parameter of interest is motion of the airfoil with respect to the platform, not the motion with respect to the disk or firtree, as considered in the early analytical studies.

The shift in emphasis of the program from the whole blade mode to the airfoil-alone mode caused changes in the experimental and analytical procedures, as described in the sections which follow.

4.1 EXPERIMENTAL EFFORT

The experimental effort conducted over the course of the program involved high speed spin testing of three test specimens instrumented with strain gages and supplemental laboratory investigations in support of the spin tests. The spin series included the following tests:

1. Spin pit testing of a solid dummy disk designed to simulate a bladed HPFTP disk assembly, to evaluate: strain gage installation methods; the planned magnetic vibration excitation method; and the operation of the spin test mechanism and associated strain circuit-slipring assembly.

2. Spin testing of a bladed disk assembly while attempting to vibrate the turbine blades by imposing magnetic force pulses on the disk. This attempt was unsuccessful and ended when the spin shaft fractured during a high speed spin.

3. Spin testing of a second bladed disk specimen utilizing magnetic pulses imposed on the blade tips to induce vibration in the blades. This test series demonstrated the vibration characteristics of the HPFTP turbine blades as affected by platform friction dampers.

Supplemental experimental efforts included the modal testing of blades and disks and the development of magnetic excitation methods and hardware

The experimental work began with the design of a test disk to carry the HPFTP blades. Heat-treated Titanium 6AL4Z was selected by UDRI as the material for the test disk because it would be lighter-weight and more flexible than steel, and would have more nearly the same elastic modulus as the HPFTP disk. The disk was designed with 64 firtree slots rather than the 63 slots in the HPFTP disk in order to provide a symmetrical test specimen having test octants containing eight blades each. Eight blade-damper test configurations or four configurations paired in octant sections across the disk diameter could be accommodated by this design. This consideration required the test disk to be 64/63 times the diameter of the HPFTP disk.

The outboard section of the test disk was configured identically to the HPFTP disk except for the diameter adjustment and extra slot. The inboard section was a hub designed by Tech Development, Inc. (TDI) which mated to a spin shaft disk drive arbor designed by ASI. A stress analysis for the test disk turning at 38,000 rpm with 64 HPFTP turbine blades installed is shown in Table 1. This analysis shows a comfortable margin of safety for the 160 ksi disk material.

4.1.1 Dummy Disk Spin Test

While TDI was fabricating the test disk, ASI machined an unslotted dummy disk having a rim area weighted to simulate the mass of the 64 test turbine blades. This disk was used for trial spins to test the spin pit shaft-arbor-disk arrangement, the planned strain gage-slipring instrumentation circuitry, and a magnetic disk vibration excitation system. These tasks were accomplished by instrumenting the dummy disk with strain gages, installing it in the spin pit with the magnetic vibration excitation system, and conducting spin pit runs while recording data just as planned for an actual test spin.

The strain instrumentation on the dummy disk included both radial gages on the disk faces and circumferential gages on the disk rim. It included both radial and circumferential wires cemented to the disk and gage leadwire terminals installed near the disk hub and near the rim. When high speed spin tests were conducted the only problems encountered with the strain gage circuitry were disbonding of circumferentially routed leadwires on the outer circumference of the disk due to the high centrifugal force there and the fact that the adhesive bonds were loaded in peel rather than in shear. It was decided then that a slot would be required in the platforms of instrumented blades since gage wires on the top surface of the platforms would be at nearly the same spin radius and would be loaded in the peel direction by centrifugal force. Gages and terminals were installed with Micro Measurements (MM) 610 or AE-15 epoxy cement and leadwires were bonded down with MM GA-61 cement. Those materials all proved to be satisfactory and were used later for the strain gage circuitry on the bladed disk test assemblies. The strain instrumentation circuitry installed on the dummy disk is shown in Figure 2.

TABLE 1

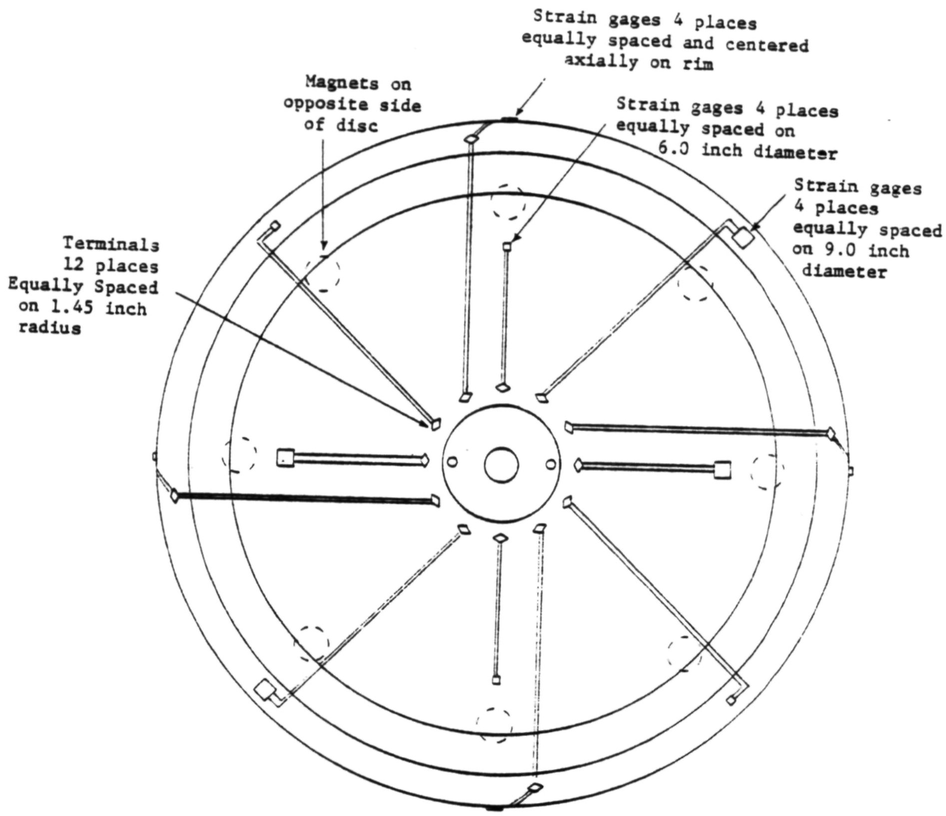
ROTATING DISK STRESSES

TECH DEVELOPMENT INC

U.D. ROTOR
38000 RPM

RADIUS INCH	THICK INCH	RHO ---	POIS RAT.	MODULUS ELASTIC	COEF. OF THER. EX	DELTA TEMP.	S-RAD PSI	S-TAN PSI
0.250	2.160	.160	.352	16500000	.0000053	0	0	104587
0.289	2.200	.160	.352	16500000	.0000053	0	13006	91019
0.328	2.200	.160	.352	16500000	.0000053	0	21677	82098
0.367	2.200	.160	.352	16500000	.0000053	0	27683	75893
0.446	2.200	.160	.352	16500000	.0000053	0	35302	67682
0.500	2.180	.160	.352	16500000	.0000053	0	38783	64388
0.570	2.000	.160	.352	16500000	.0000053	0	45000	62685
0.640	1.880	.160	.352	16500000	.0000053	0	49394	61843
0.800	1.740	.160	.352	16500000	.0000053	0	54747	60804
0.880	1.640	.160	.352	16500000	.0000053	0	58093	61371
0.960	1.470	.160	.352	16500000	.0000053	0	64418	63451
1.000	1.340	.160	.352	16500000	.0000053	0	70283	65667
1.040	1.260	.160	.352	16500000	.0000053	0	74238	67363
1.100	1.160	.160	.352	16500000	.0000053	0	79715	69920
1.150	1.100	.160	.352	16500000	.0000053	0	83201	71783
1.200	1.050	.160	.352	16500000	.0000053	0	86243	73548
1.300	0.980	.160	.352	16500000	.0000053	0	90451	76471
1.450	0.900	.160	.352	16500000	.0000053	0	95428	80367
1.700	0.820	.160	.352	16500000	.0000053	0	99564	85024
2.000	0.760	.160	.352	16500000	.0000053	0	101337	88667
2.250	0.720	.160	.352	16500000	.0000053	0	101939	90787
2.500	0.700	.160	.352	16500000	.0000053	0	99848	91456
2.750	0.680	.160	.352	16500000	.0000053	0	97710	91644
3.000	0.660	.160	.352	16500000	.0000053	0	95434	91440
3.180	0.660	.160	.352	16500000	.0000053	0	91625	90312
3.300	0.650	.160	.352	16500000	.0000053	0	90429	89936
3.400	0.650	.160	.352	16500000	.0000053	0	88236	89156
3.480	0.650	.160	.352	16500000	.0000053	0	86464	88486
3.560	0.650	.160	.352	16500000	.0000053	0	84673	87777
3.620	0.690	.160	.352	16500000	.0000053	0	78474	85481
3.670	0.770	.160	.352	16500000	.0000053	0	69317	82075
3.700	0.980	.160	.352	16500000	.0000053	0	53947	76471
3.750	0.980	.160	.352	16500000	.0000053	0	53028	75737
3.800	0.980	.160	.352	16500000	.0000053	0	52091	74999
3.910	0.980	.160	.352	16500000	.0000053	0	49968	73358

NUMBER OF BUCKETS = 64
WEIGHT OF BUCKETS = 0.1
RADIAL DISTANCE TO C.G. = 4.582



Note: Gages are radially aligned with magnet centers

□ Strain gage, type EA-09-062 AP-120

▫ Strain gage, type CEA-09-062 UW-120

◇ Terminals, type CEG-25C

Figure 2 Dummy Disk Magnet and Strain Gage Installations

The strain instrumentation circuits were all installed on the lower or aft surface of the test disks and on the lower surface of the blade necks and airfoils. High magnetic strength 0.5 inch diameter, rare earth permanent magnets were installed on the upper surface of the dummy disk and first test disk for the purpose of exciting resonance modes in the disk to drive the blades into vibration. As shown in Figure 2, eight magnets were installed at evenly spaced locations near the rim of the disk. It was planned to apply force pulses to these magnets either in alternate directions on alternate magnets to excite the 4N mode of the disk or in the same direction on all eight magnets to excite the umbrella mode of the disk. Modal analyses of the dummy disk and slotted test disk had shown that both of these modes occurred near the first mode flexural resonance frequency of the HPFTP blades at 4500 Hz. Eight direct current solenoid type electromagnets were fabricated to act on the permanent magnets. The electromagnets were attached to a plate which was bolted to the spin pit lid. They were aligned parallel to the spin shaft axis to apply transverse force pulses to the disk through the reaction of the permanent magnets as they passed through the electromagnet fields when the disk was spinning. The electromagnet assembly is shown in Figure 3.

The trial spins of the dummy disk also were used to check out the spin system and the high speed slipring circuitry. The spin system arrangement is shown in Figure 4. In this arrangement the strain circuit leadwires located on the bottom of the disk are routed through the hollow arbor and quill shafts to the slipring set at the top of the spin system and then on through strain gage amplifiers to an Ampex FR 1300 FM recorder.

Two disturbing things happened during the trial spins of the dummy disk. The first was a very high voltage induced in the strain gage circuits as they passed through the fields of the electromagnets. The induced voltage was orders of magnitude higher than the expected strain signals and no vibration strains could be detected. The second disappointment was the fact that during the trial spin two of the permanent magnets from the disk were found attached to electromagnets after unbonding from the disk, evidently during spindown.

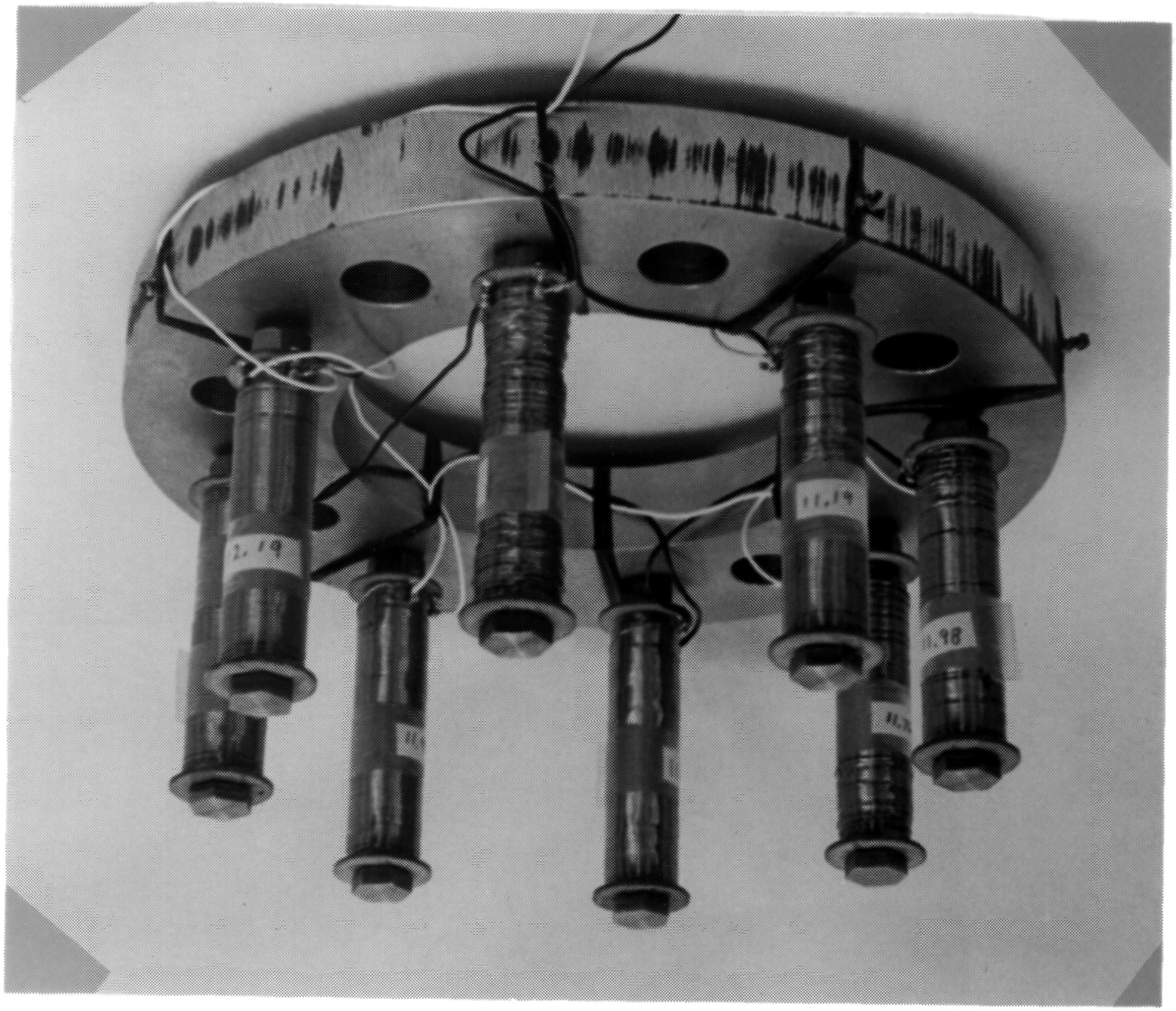


Figure 3 Electromagnet Assembly for Disk Transverse Excitation

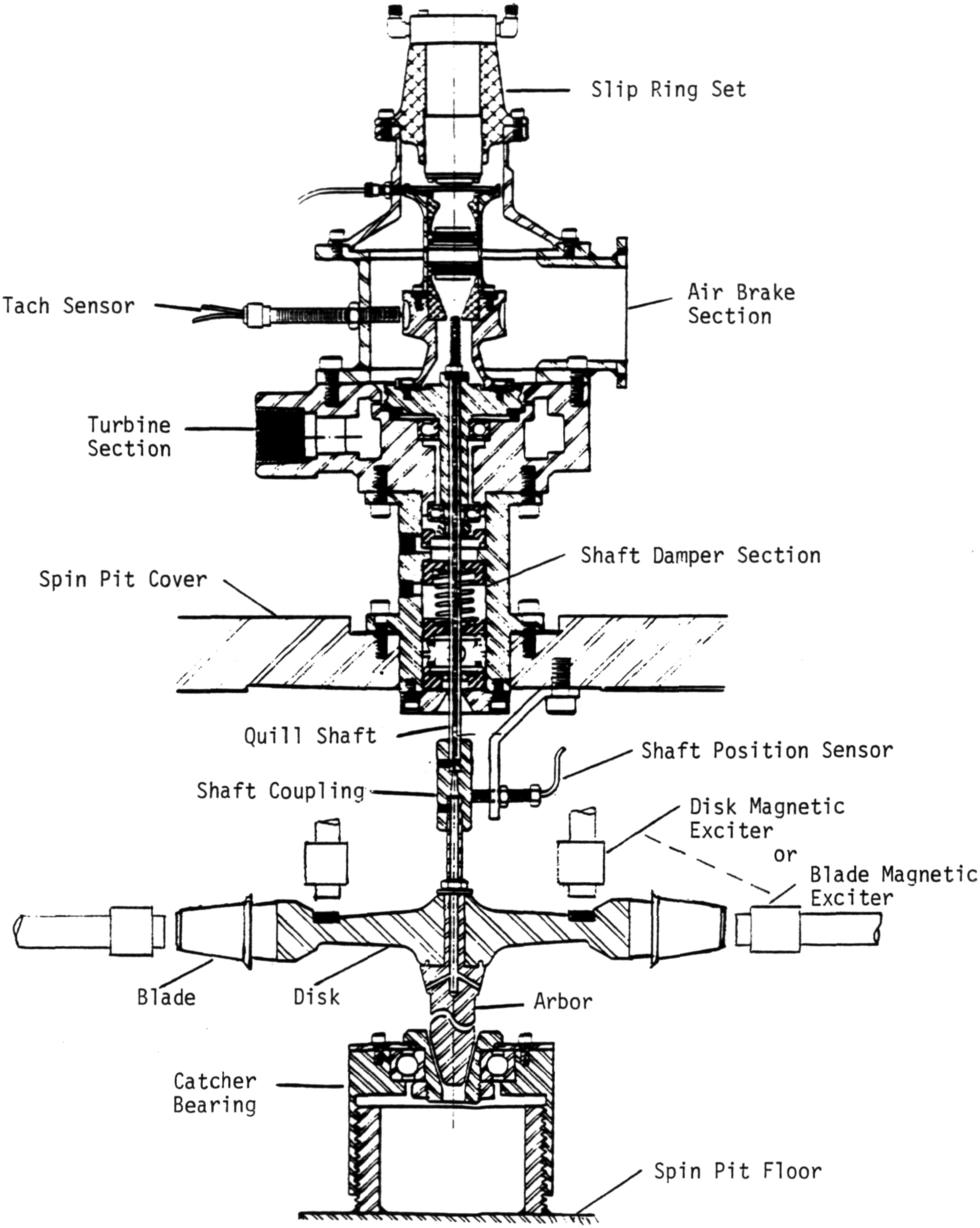


Figure 4 Spin System Arrangement

It was decided then to use permanent magnets for the fixed as well as the disk magnets since they could be mounted so as to have a much shallower depth of field than the solenoid electromagnets. This would greatly reduce the voltage induced in the strain circuits on the bottom side of the disk. The unbonding of the permanent magnets from the disk was attributed to heating of the magnets by the pulses caused as they passed through the electromagnet fields. Several high temperature adhesives then were tested and a polyamide cement with twice the strength at 500°F of MM GA-61 epoxy was selected for use in installing the magnets on the test disk.

Some other facts revealed during the trial spins were that the slipring system worked well in spins to 35,000 rpm and that more clearance than expected was required in the setup of the tapered catcher bearing because of deflection of the lid and base of the spin pit by the pressure differential when a good vacuum was pulled in the pit. The spin system and spin shaft position monitors worked quite well.

4.1.2 First Test Specimen - Disk Excitation

While the trial spins of the dummy disk were being completed in late 1982, work was already underway on the bladed disk test specimen. That work included the installation of strain gages and leadwires on turbine blades and on the test disk as well as modal studies of several blades and the disk. Then design and fabrication of experimental friction dampers and of a new magnetic excitation system and its mounting hardware were required.

Sixty-four used HPFTP turbine blades were furnished for use in the test program. A set of new production dampers (0.56 gram) also was furnished. Sixteen of the test turbine blades were instrumented with 1/16 inch square strain gages on the suction side of the airfoil. The gages were centered 1/4 inch above the platform and 1/8 inch from the trailing edge, a high stress region of the blade. Leadwires were routed down the aft face of the blade neck through a slot cut in the platform to terminals on the aft face of the firtree area. A photograph of a strain gage installation on a test blade is shown in Figure 5.

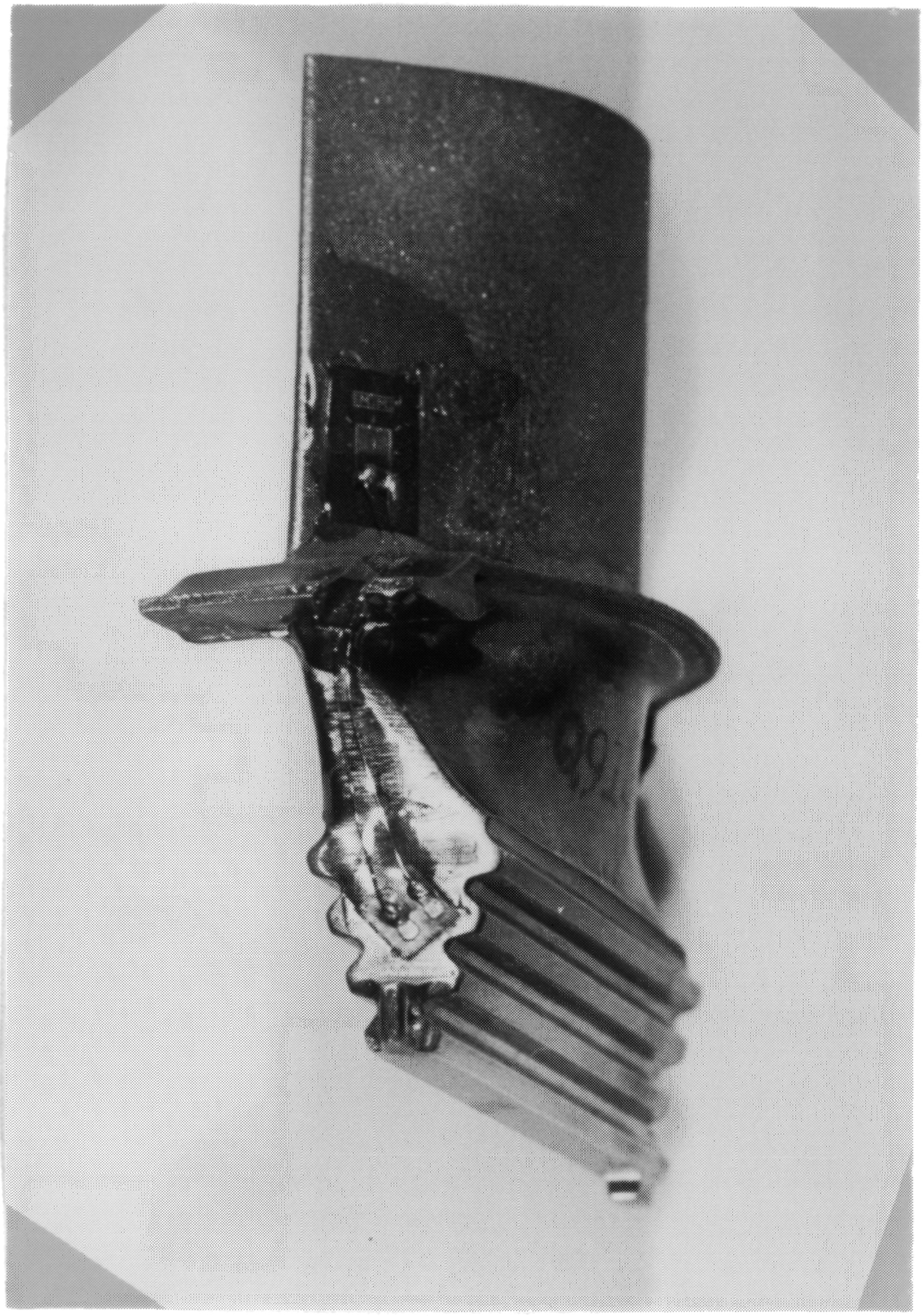


Figure 5 HPFTP Blade With Wired and Sealed Strain Gage

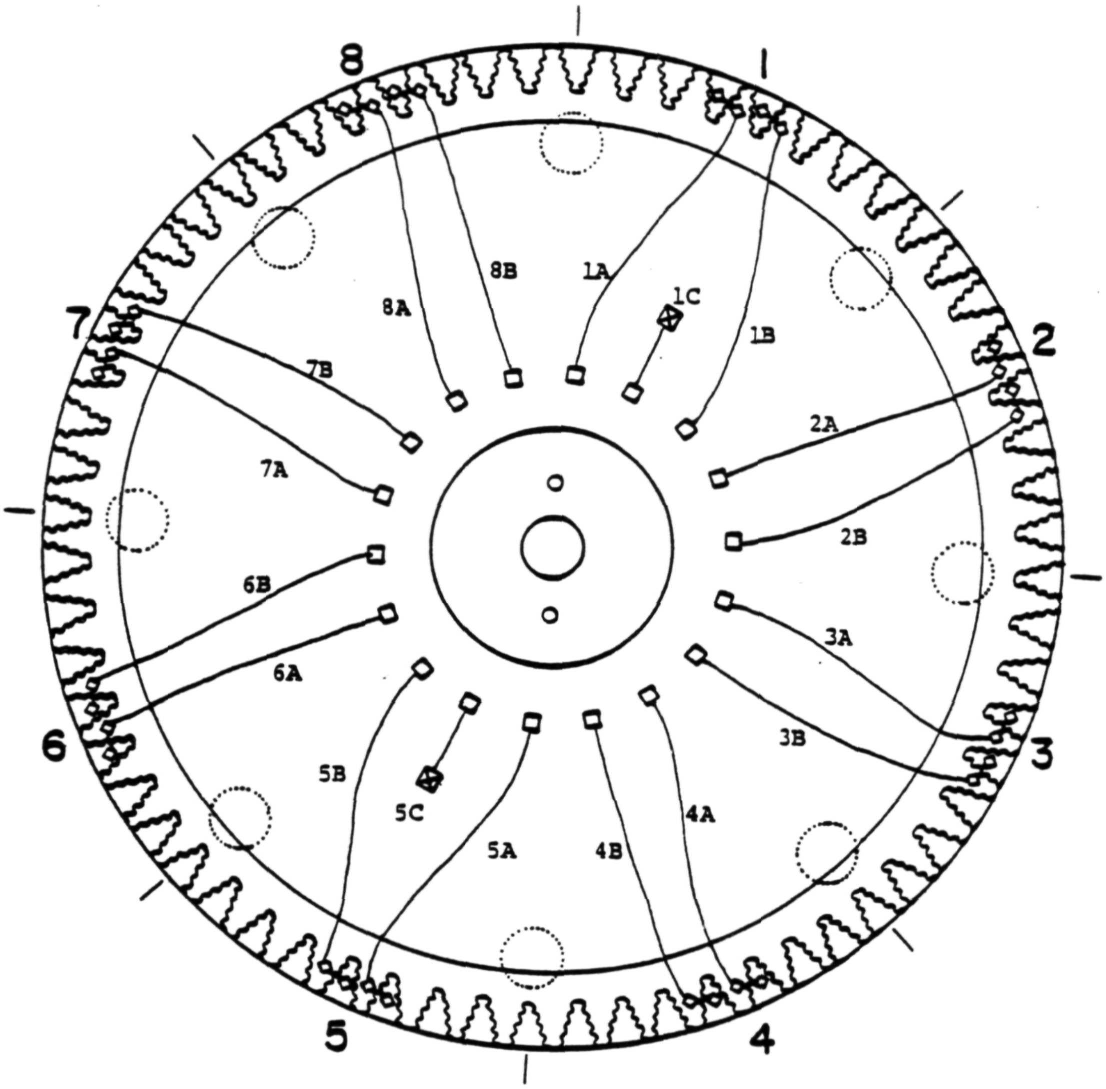
Strain gage leadwire circuits for the sixteen instrumented blades and for two radial strain gages located at high stress areas of the test disk were installed to the bottom surface of the disk as shown in Figure 6. This figure also shows the locations of the eight permanent magnets installed near the rim on the top surface of the disk and lists the damper configurations for the eight test octants of the bladed disk assembly. Two strain gaged blades were located as the center blades of each test octant of eight blades.

The eight test octants were configured with four pairs of duplicate installations balanced across the center of the disk. One pair had no dampers installed and the other three pairs had production 0.56 gram dampers, 0.20 gram nichrome wire dampers, and 0.10 gram nichrome wire dampers, respectively. The experimental wire dampers were formed on bending jigs designed for that purpose. The three damper types are shown in Figure 7.

As a result of the high voltages induced in the strain circuits during the trial spins by the magnets aligned transversely to the disk, UDRI proposed adding magnetic material to the blade tips and exciting the blades directly with permanent magnets aligned radially to the disk. In this way the magnets would be a greater distance from the strain circuits, the magnets could be installed with alternating polarities so that induced voltages would be separated in frequency from induced force pulses, and various numbers of magnets could be installed to excite the low order blade resonance modes at various spin speeds. This concept was accepted but it was decided to attempt the disk transverse excitation method first. Accordingly, the fixed magnet mounting fixture shown in Figure 8 was designed and fabricated. The top plate of this fixture was supported from the spin pit lid and could hold eight permanent magnets in their holders to excite the disk in the transverse direction by interaction with the identical permanent magnets installed on the disk. The cylindrical sleeve bolted to the top plate and could be fitted with twenty-eight, fourteen, or eight magnets located symmetrically around its lower circumference to interact with magnetically permeable material attached to the sixty-four test

OCTANTS

1,5 - No Damper
2,6 - .56 gm Prod Dampers
3,7 - .10 gm Wire Dampers
4,8 - .20 gm Wire Dampers



Terminal, Type CEG-25C
Terminal, Type CEG-38C
Strain Gage, Type EA-09-062AP-120

Figure 6 First Test Disk-Strain Circuits, Dampers, and Magnets

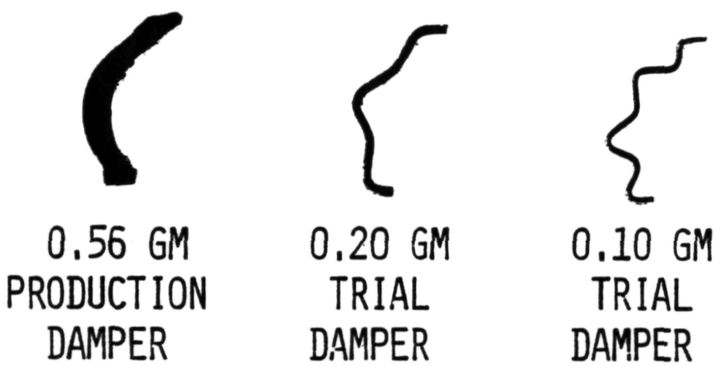


Figure 7 Test Damper Types

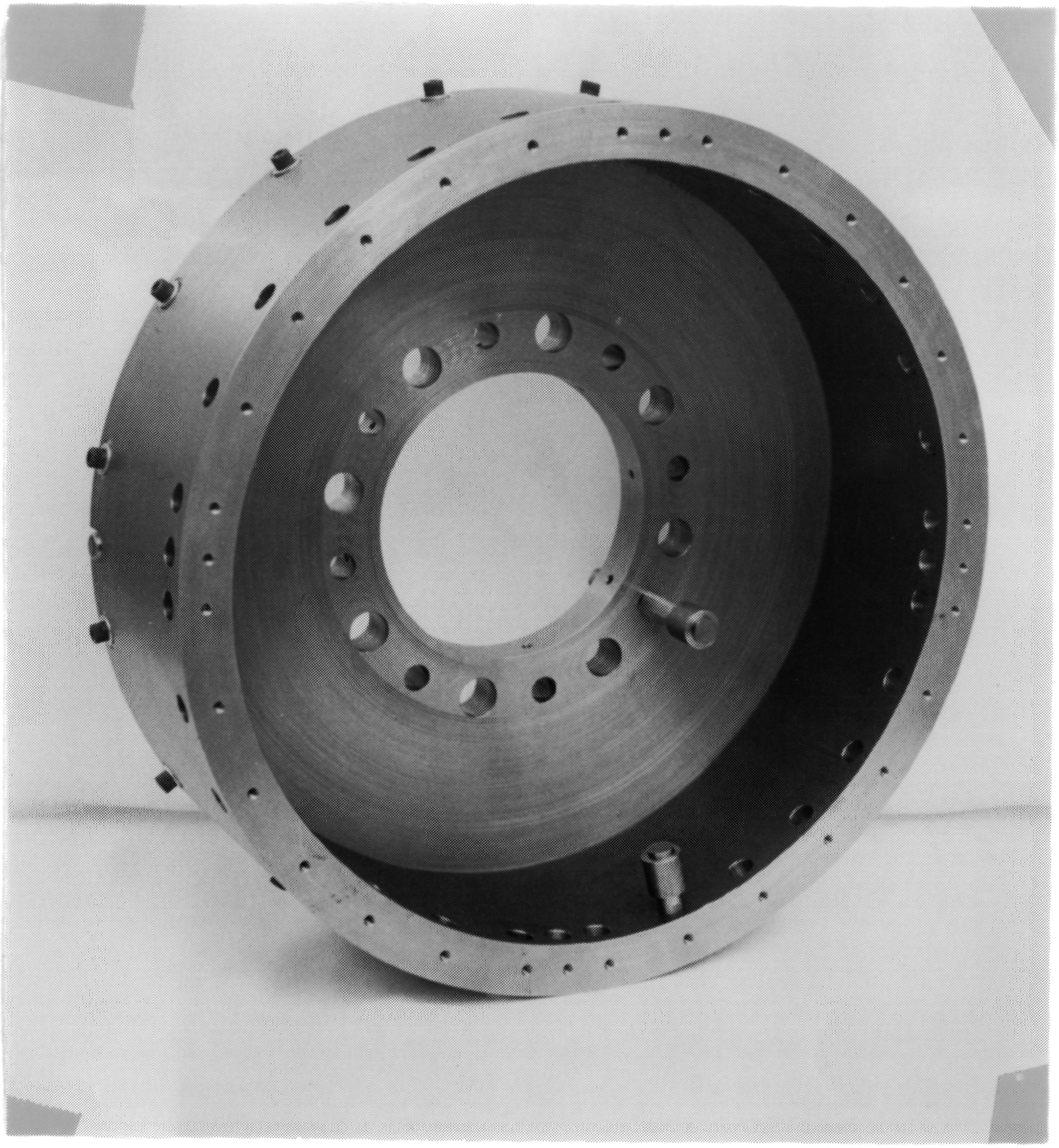


Figure 8 Excitation Magnet Support Fixture

blade tips. The magnet holders were adjustable to allow variation in the gaps between the fixed magnets and either the disk magnets or the blade tips.

Electroplated nickel and cobalt weld beads were considered for use as the magnetic material to be added to the blade tips. Cobalt weld beads were selected because of the higher magnetic permeability and the higher Curie temperature of that material. Metallurgical examinations and stress analysis studies showed the cobalt weld beads would easily sustain the centrifugal loading at the blade tips. Vibration tests of several blades showed that the resonance frequency of the first flexural mode of the blades was not affected significantly by the 0.12 gram of cobalt welded to their tips. Two views of an installed weld bead are shown in Figure 9.

After the strain gages and weld beads were installed to the blades, they were weighed precisely and scheduled into the disk for optimum balance of the assembly. The blades and appropriate dampers then were installed in the disk firtree slots and the jumpers were installed between the blade and disk strain gage circuit terminals at the firtree area. The top or forward side of the completed test specimen is shown in Figure 10. The bottom or aft side of the specimen is shown in Figure 11.

When this stage of assembly was completed the test specimen was delivered to ASI for installation to the drive arbor and then installation of the strain gage leads which ran from the disk into the hollow arbor and quill shafts to the slipring assembly mounted on top of the spin pit drive turbine. After installation to the arbor and installation of the gage leads the spin test assembly was dynamically balanced to 0.02 gram-inches at 3000 rpm by the Balancing Company (BalCo) of Vandalia, Ohio. The gage leads then were routed through the quill shaft to the slip ring connector by ASI and the final spin test assembly was completed as shown schematically in Figure 4 (page 13) and photographically in Figure 12.

With the decision made to utilize disk excitation for the first spin test, a further decision was required as to which disk mode to

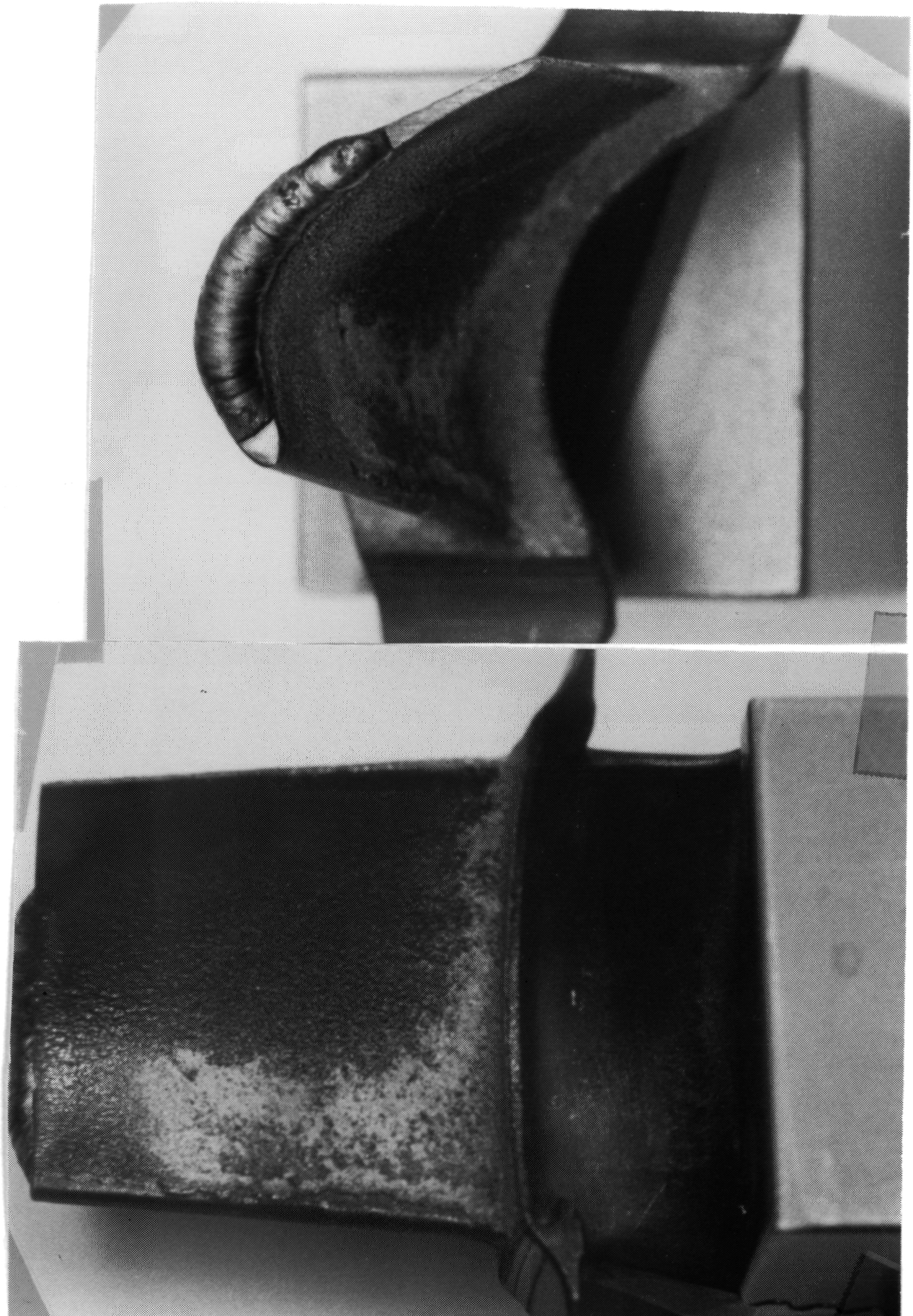


Figure 9 Cobalt Weld Beads on Blade Tips

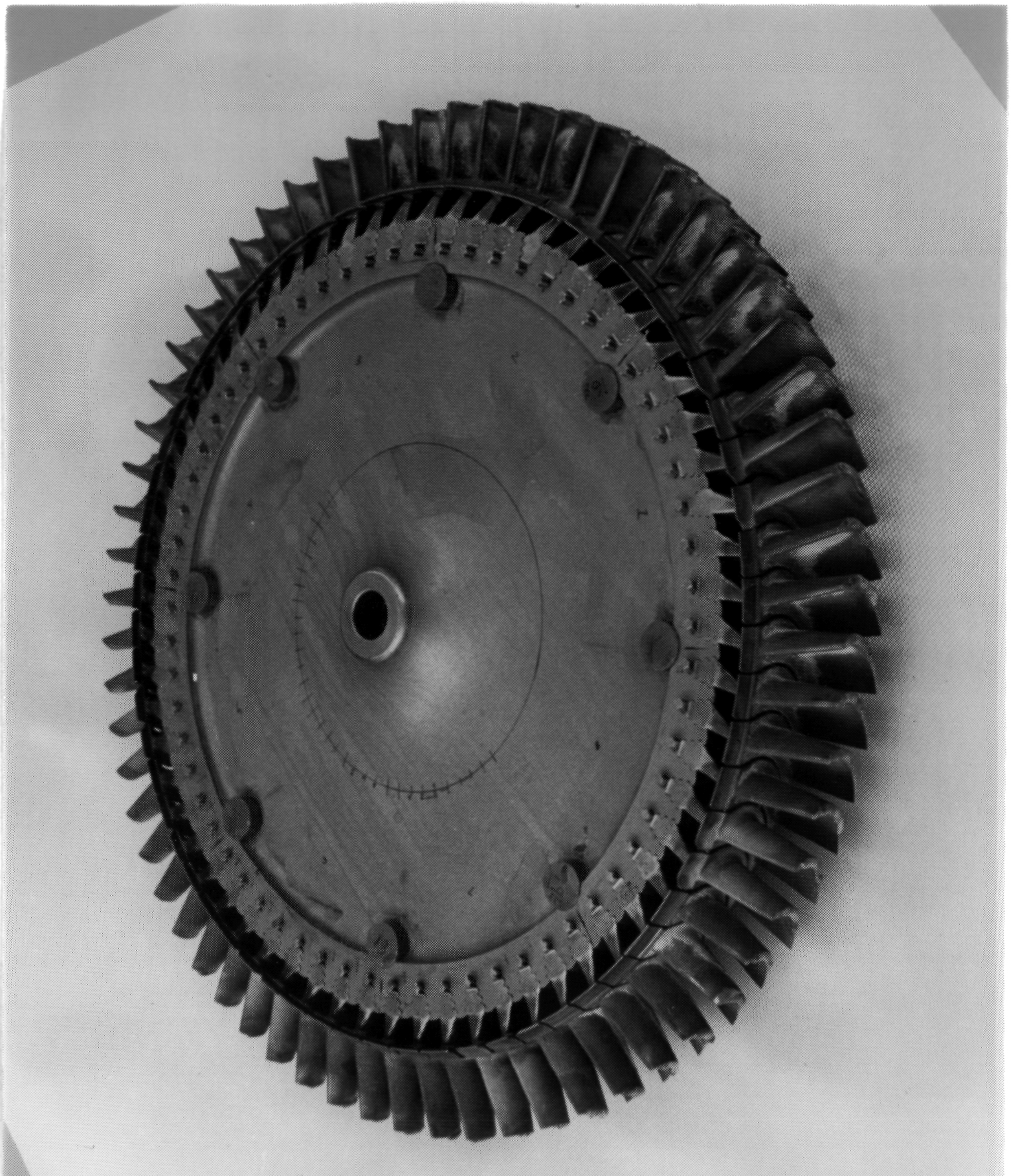


Figure 10 First Test Disk-Top or Forward Side

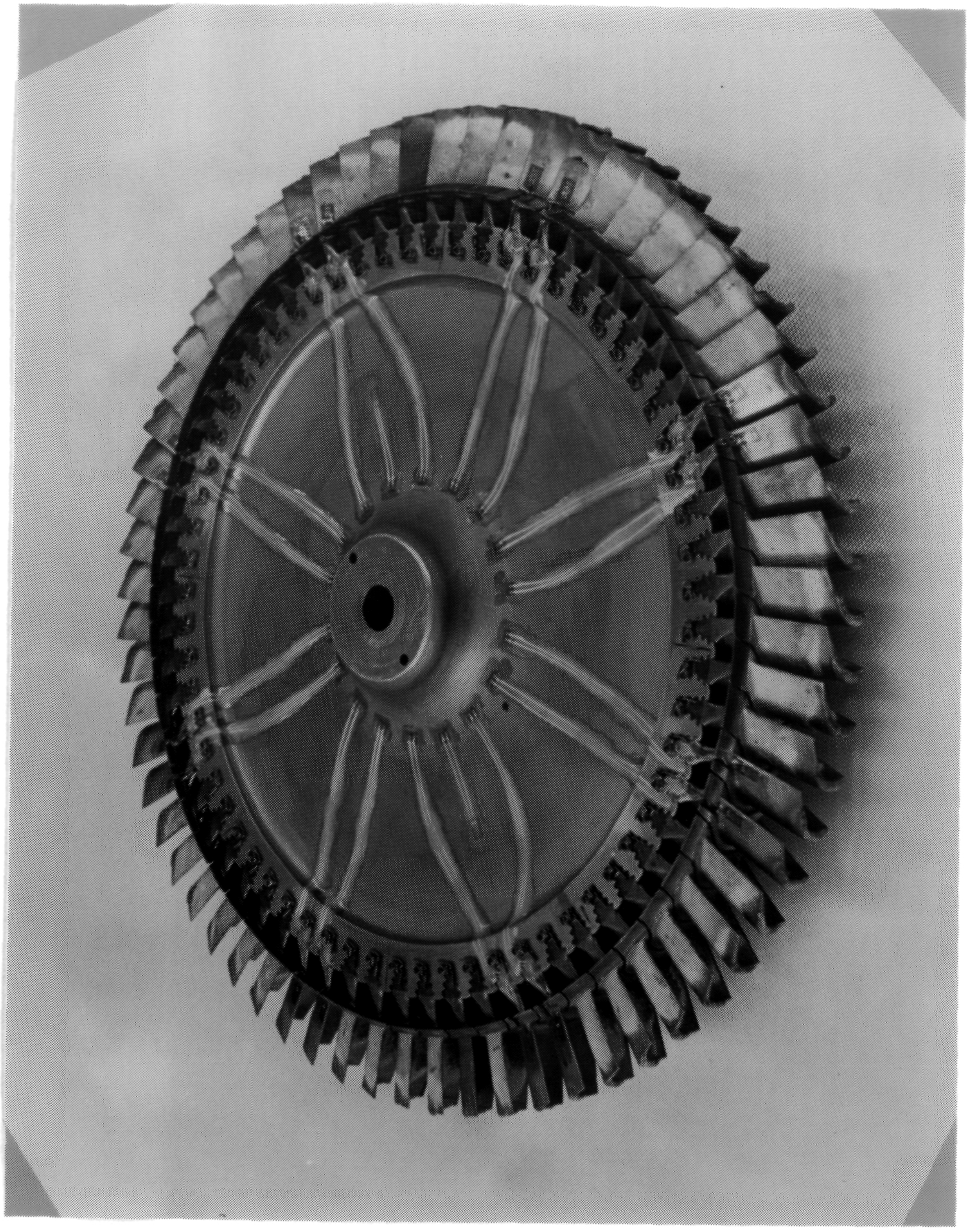


Figure 11 First Test Disk-Bottom or Aft Side

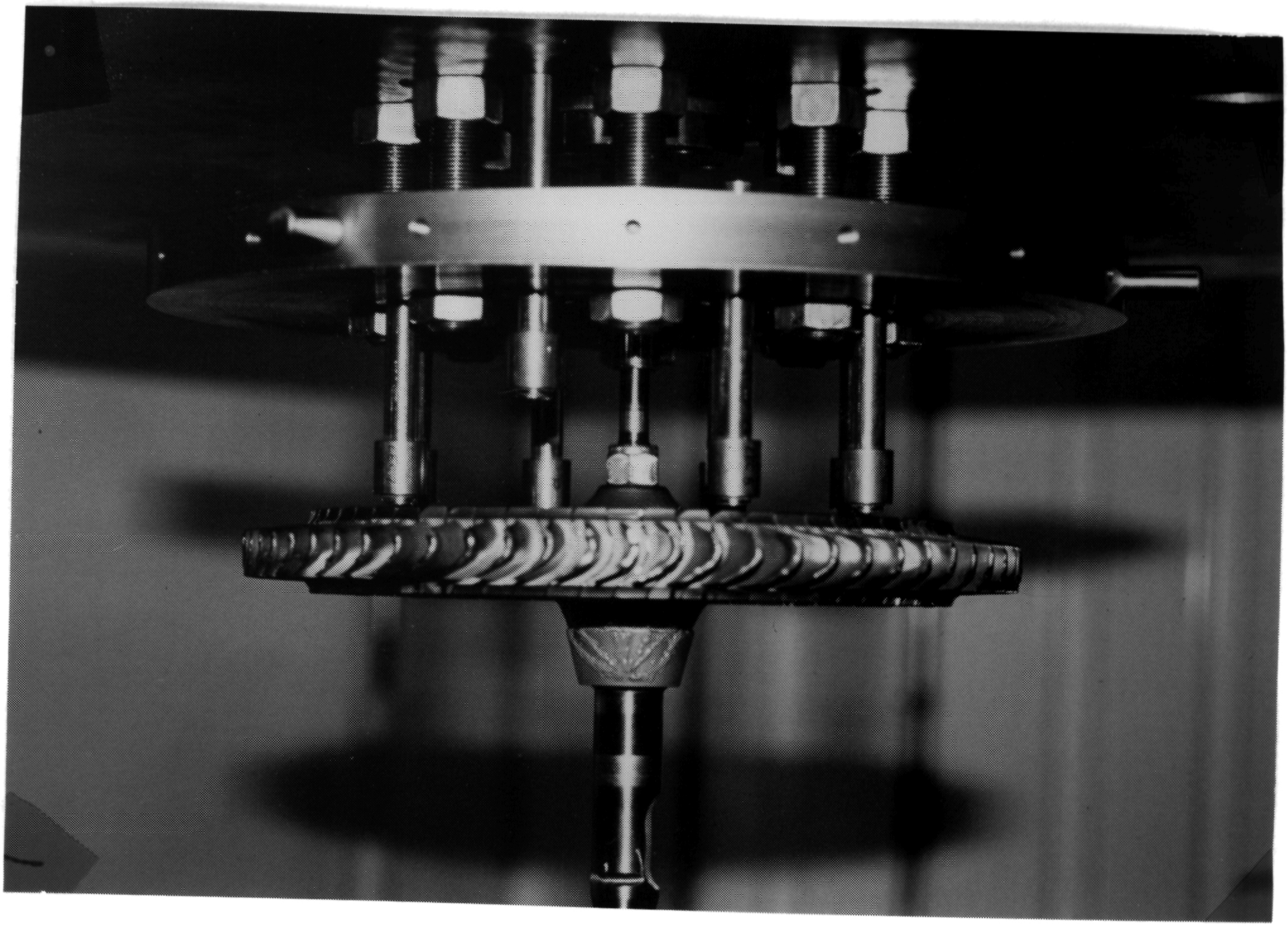


Figure 12 First Test Disk Setup On Spin Assembly

excite. The disk modal analysis had shown that the umbrella mode occurred at 3925 Hz and the 4D mode occurred at 4475 Hz for the bare disk at static conditions. It was assumed that centrifugal loading during high speed spin would increase these frequencies significantly and that the umbrella mode was more likely to coincide with the 4500 Hz flexure mode of the blades than the 4D disk mode. Also, the 4D mode would have to be excited by harmonics of the excitation pulse train caused by oppositely polarized fixed magnet adjacent pairs since the fundamental would not reach 4500 Hz until the spin speed reached 4500/4 x 60 or 67,500 rpm. The umbrella mode could be reached at about half that spin speed since the magnet polarities would be arranged to produce eight unipolar pulses per rev at the eight disk magnet locations. Accordingly, the eight fixed magnets and eight disk magnets were mounted with like magnetic poles facing each other. This would produce repulsion forces as the fixed and disk mounted magnets passed each other during disk spins.

The prospect of using all repulsion forces for the magnet interactions was attractive for two reasons. First, repulsion forces would help to hold the magnets mounted on the spinning disk in their shallow sockets if the polyamide cement used to hold them was weakened by heating of the magnets. Second, the repulsion forces would tend to stabilize the spinning disk in a precise rotational plane. Any deviation from planar rotation caused by whirl of the system or wobble of the disk would result in the generation of restoring forces by the temporary asymmetry of the magnet system. This can be deduced easily from the graph of magnet interaction forces vs magnet separation distance shown in Figure 13. The high gradient of force vs distance would produce substantial restoring forces at the planned magnet gap distance of 3/16 inch (4.75 mm). The nominal force generated at magnet passage coincidence at that separation distance can be seen to be approximately one pound (450 gm) at each magnet. The excitation force on the disk thus would be eight pounds imposed eight times per disk revolution and exerted as eight one pound pulses equally spaced around the rim of the disk. The forced disk vibration was intended to excite the first bending mode of the turbine blades (4400 to 4700 Hz) at an 8E (eight per rev) frequency (equivalent to 33,000 to 35,250 rpm).

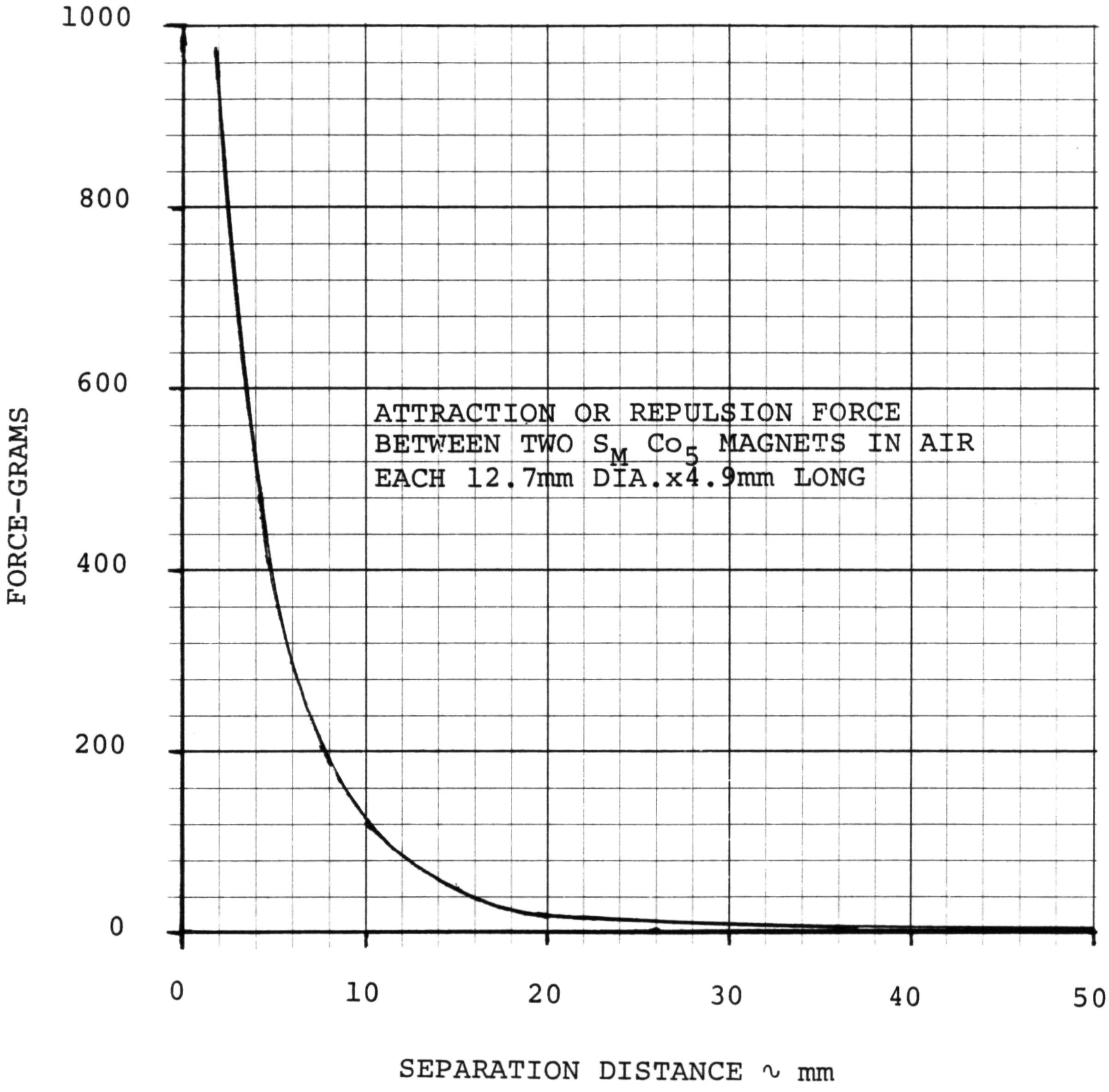


Figure 13 Permanent Magnet Interaction Forces

On the morning of 27 April, 1983, the disk was rotated at low speed (2000 rpm). No balance or alignment problems were detected. The shaft position sensors showed no shaft wobble. The tapered catcher bearing was set at 3/16 inch radial clearance around the lower arbor conical end. The strain gage circuits were checked and found to be isolated and within nominal gage resistance tolerances. The leadwires and sliprings added two ohms to the 120 ohm gage resistance valves giving a desensitization of 1.67 percent for the two-wire single active arm gages. Ten gages were connected to strain amplifiers, were balanced, and calibration levels were set. One gage was monitored as a voltage source to measure EMF pulses generated by passages through the fields of the eight fixed excitation magnets. The eleven strain gage amplifier signal outputs were routed to an Ampex FR 1300 wideband FM magnetic tape recorder. The spin pit tachometer signal and voice microphone were connected to two additional channels of the recorder. The tachometer, spin shaft displacement monitors, spin pit absolute pressure, and a thermocouple from an excitation magnet holder were displayed on meters on the spin pit control console. The strain gages recorded were as follows (see Figure 6):

Tape Track	Location	Gage No.	Data
1	Blade 4	1A	Instanteous Strain (DC to 10 KHz)
2	Blade 5	1B	" "
3	Disk	1C	" "
4	Blade 12	2A	" "
5	Blade 20	3A	" "
6	Blade 28	4A	" "
7	Blade 37	5B	" "
8	Blade 45	6B	" "
9	Blade 53	7B	" "
10	Blade 61	8B	" "
11	Blade 13	2B	Magnetic EMF Pulses

After noon on 27 April, 1983 the spin pit vacuum pump was started, strain circuit balances were checked, and the test beginning

calibrations were recorded on the strain gage channels. Low speed spin and data recording started with the spin pit absolute pressure at 3 torr (mm Hg). The pressure reduced to 2.5 torr during spin. A steady acceleration of spin speed reached 25,000 rpm in approximately 3 minutes. The spin shaft was absolutely stable with no measureable deflection shown by the spin shaft position monitor. Only a minor temperature rise of approximately 20°F was shown by the magnet holder temperature monitor. At 25,000 rpm the spin acceleration rate was halved. A spin speed of 32,000 rpm was reached smoothly and without incident. Magnetholder temperature was 110°F. No spin shaft deflection occurred. About 5 seconds after 32,000 rpm was reached a boom lasting less than a second was heard from the spin pit. The strain gages open-circuited instantaneously, and the spin turbine was stopped very quickly. It was later determined that the spin was almost exactly 32,500 rpm when the failure incident occurred.

When the spin pit lid was removed the scene shown in Figures 14 and 15 was revealed. The arbor shaft was broken in two places and the test specimen had impacted the sidewall. The bladed disk had made about two revolutions around the spin pit sidewall before its spin momentum was dissipated. All 64 blades were broken off either in the neck or upper firtree area, except for four which fractured above the platform. All eight disk magnets were knocked off, and most of them came off through a combination of magnet fracture and adhesive fracture. Figure 16 shows two magnet sockets and several blade stubs after the failure. Figure 17 shows the lower failure of the arbor spin shaft. This was judged to be a secondary shaft failure. Figure 18 shows the condition of the upper arbor stub, the shaft coupling, and the primary arbor shaft failure in the thread relief below the disk lock nut thread.

The two shaft failures were examined carefully and were judged to be due to a combination of torsional and bending loadings. No fatigue striations or defects in the shaft material were found. A small area of brittle fracture was found in the upper or primary fracture. This was judged to be an area of plane strain fracture during the instantaneous failure. The probable failure sequence was estimated to be:

Figure 14 Spin Pit After Shaft Failure

Figure 15 Spin Pit After Shaft Failure

Figure 16 Magnet and Blade Sockets After Failure

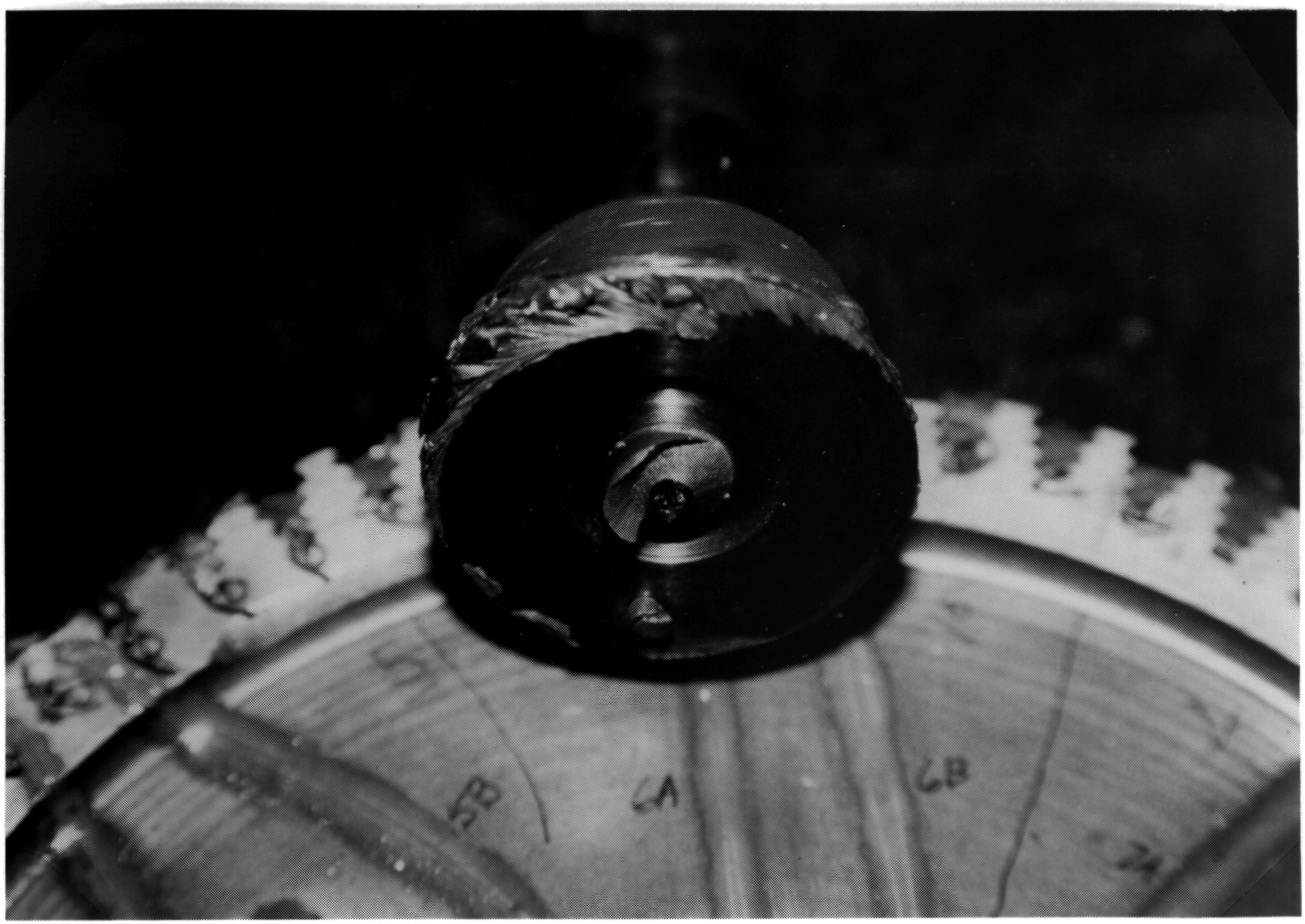


Figure 17 Lower Arbor Shaft Failure

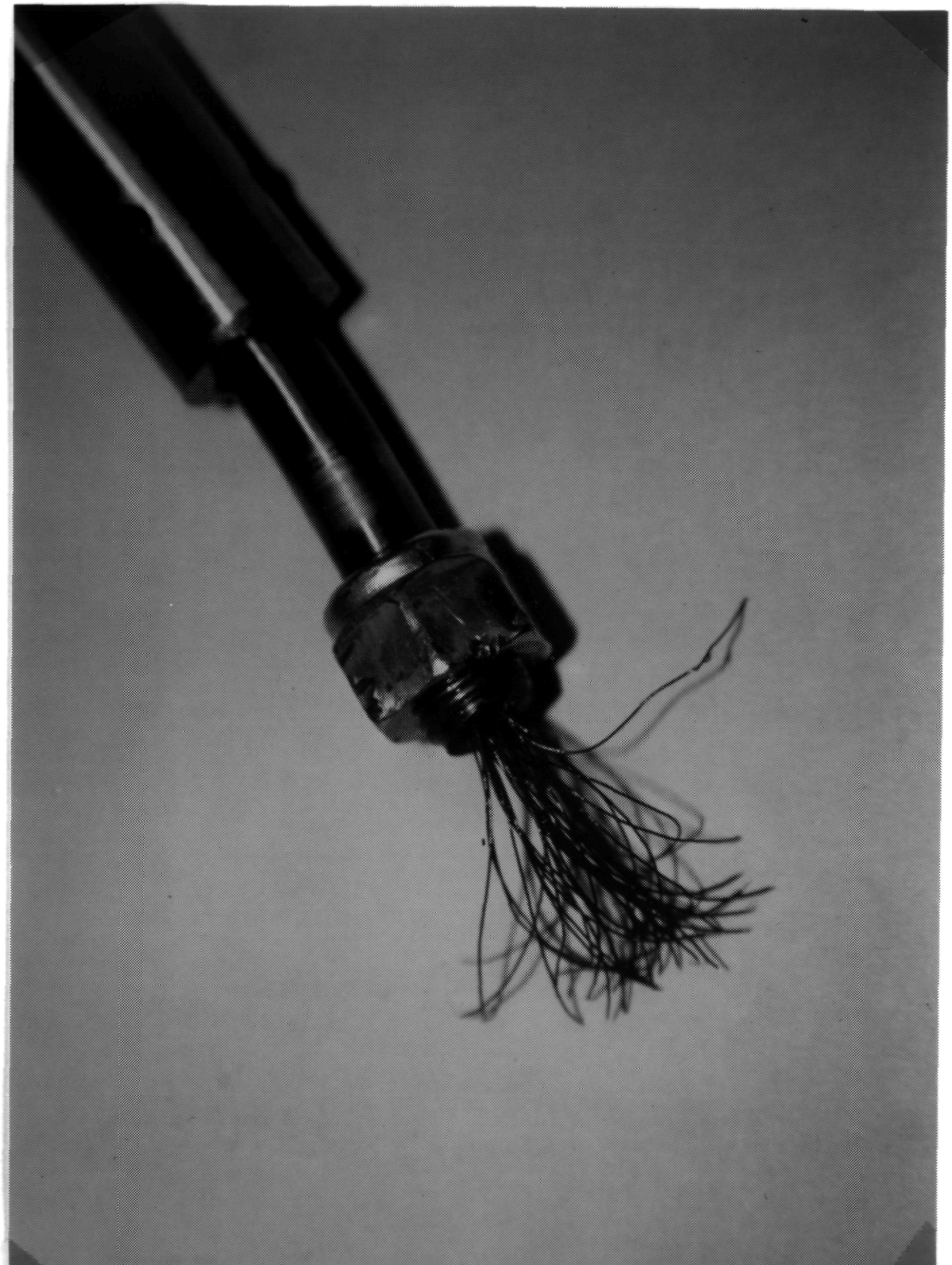


Figure 18 Primary Arbor Shaft Failure

first, a loss of mass from a single location on the specimen; second, an instantaneous side load deflecting the shaft and causing engagement of the arbor with the stationary catcher bearing; third, the combination of the side load and the torsion load due to the catcher bearing inertia fracture the shaft at the lock nut thread relief; fourth, the lower shaft fracture occurs and the side load drives the disk into the spin pit sidewall; and fifth, the arbor comes out of the catcher bearing and impacts the wall at the penetration hole shown at the left center of Figure 15. The location of the wall impact of the first mass ejected was not found, probably because it occurred in the area sawed out by the disk derotation, nor was the ejected mass found. It may have been a magnet, a weld bead, a piece of epoxy, or a piece of a blade. It would seem that a mass as large as a magnet or a piece of blade would be required to cause the rather large side load that must have occurred. Ejection of a 5 gram magnet at 32,500 rpm would have caused a side load of approximately 1000 pounds.

Careful and repeated examination of the recorded strain gage data showed simultaneous and instantaneous failure of all eleven gage circuits when the shaft fractured and the strain gage leads were broken. The strain gages on the undamped blades (Blades 4,5, and 37 - Figure 6) were just beginning to show indications of approaching resonance vibration when the failure occurred. This was indicated by the presence of a beat frequency signal of low amplitude imposed on the peaks of the EMF pulses generated by passage of the gage circuits through the magnetic fields of the stationary excitation magnets. The excitation frequency when failure occurred was 4333 Hz. None of the other six blades' data sets showed indications of approaching resonance. Their platform friction dampers would be expected to raise the first resonance frequency above the 4400 to 4700 Hz first mode resonance of an undamped blade.

The test failure had destroyed the disk, blades, dampers, spin shaft position monitors, and the spin shaft. Fortunately no serious damage occurred to the spin system or slipring assembly. At NASA's request an effort was begun immediately to construct a new test

specimen to be tested with radial magnets acting to excite the blades directly by means of cobalt weld beads added to the blade tips.

4.1.3 Second Test Specimen - Blade Excitation

The second test disk also was fabricated by Tech Development, Inc. It was identical to the first disk except that it had no magnet sockets. Applied Sensors International fabricated a new quill shaft and arbor, and purchased and installed a new pair of shaft deflection sensors.

When a replacement set of HPFTP first stage turbine blades (a used set of blades, as was the first set) was received, UDRI installed cobalt weld beads to the blade tips. The entire set of blades then was x-rayed to determine that the cobalt welds were adequate and that no serious voids or cracks were present in the blades. In addition, all sixty-four test blades were tested by the impact method for first mode resonance while hard mounted in our firtree socket broach block. The blade first mode resonance frequencies found in those tests are shown in Table 2. Some unexpectedly high resonance frequencies to 5100 Hz were shown in this study, but no low resonance frequencies indicative of cracked or defective blades were found.

Twelve of the blades were instrumented with strain gages for this blade excitation spin series. Strain circuit leads and two disk strain gages were installed to the lower surface of the disk. New 0.20 gm and 0.10 gm wire dampers were fabricated. The blades were precisely weighed and scheduled in the disk for optimum balance of the test specimen. The specimen again was assembled with four damper configurations balanced in octants across the disk as shown in Figure 19.

The test disk to blade strain lead jumpers were installed and sealed, the specimen was installed to the new arbor, and the slipring leadwires were installed. The disk then was delivered to Balco for precision dynamic balance. Balco experienced problems in balancing the test specimen and a thin sacrificial balance disk was installed below the arbor lock nut on top of the test disk. The specimen then was installed in the spin pit for testing with radial blade excitation magnets, as shown in Figure 20.

TABLE 2

FIRST FLEXURAL RESONANCE FREQUENCIES
OF HPFTP FIRST STAGE BLADES

Second Blade Set

Serial Number	Frequency (Hz)	Serial Number	Frequency (Hz)
S126	4525	T326	4675
W512	4925	EV25	4775
V818	4900	W420	4825
U428	4750	V219	4900
Y42	4750	ET17	4925
U517	4850	Z819	4925
EM10	4825	BT28	4800
EM28	4950	EP7	5100
DP28A	4850	EN19	4925
Y219	4850	Y315	4825
Z414	4975	U99	4875
U628	4875	Z925	4850
EM4	5075	ER13	5050
W314	4775	V59	4775
EP11	4900	U64	4750
ER82	4750	EP20	4650
W515	4850	EM9	4625
V218	4950	EL29	4825
DL2A	4850	ER21	4850
EN6	4675	EP13	4725
Y325	4850	EN25	4950
EP29	4850	EA3	4725
X31	4750	EV28	4925
Y417	4775	33?	5000
C821	5000	T123	4700
EP18	4850	Y51	4900
U311	4950	9W23	5025
Y720	4750	U34	4725
EP22	4850	EP4	4675
V222	4775	EP12	5300
Z916	4925	EP25	4700
Y312	4825	EP28	4850

Blades from First Blade Set

Serial Number	Frequency (Hz)
D49	4550
E427	4450
None	4625 (reject blade)

Octant	Damper Type
1,5	None
2,6	0.57 gram production
3,7	0.11 gram wire
4,8	0.20 gram wire

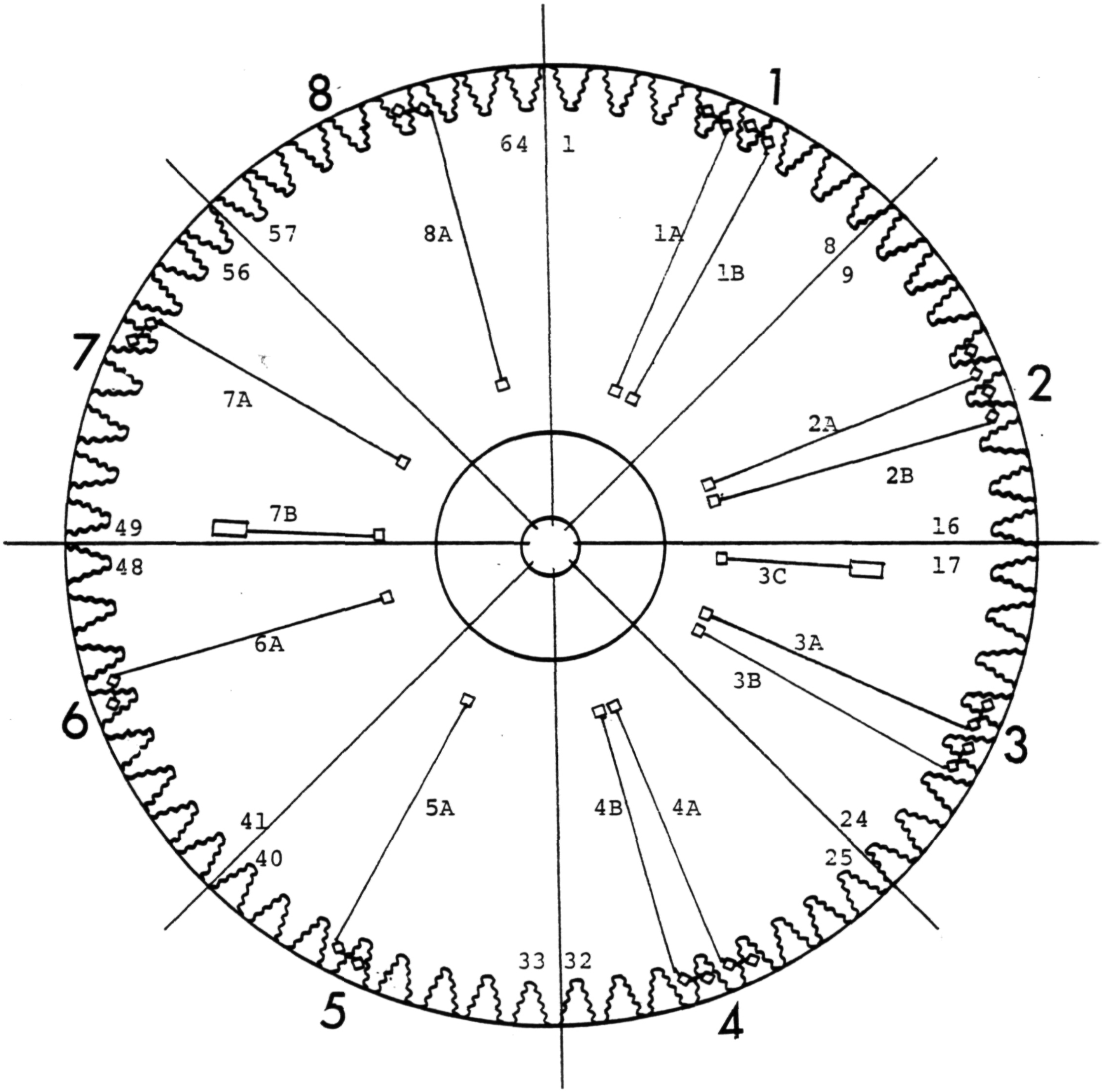


Figure 19 Second Test Disk - Strain Circuits and Dampers

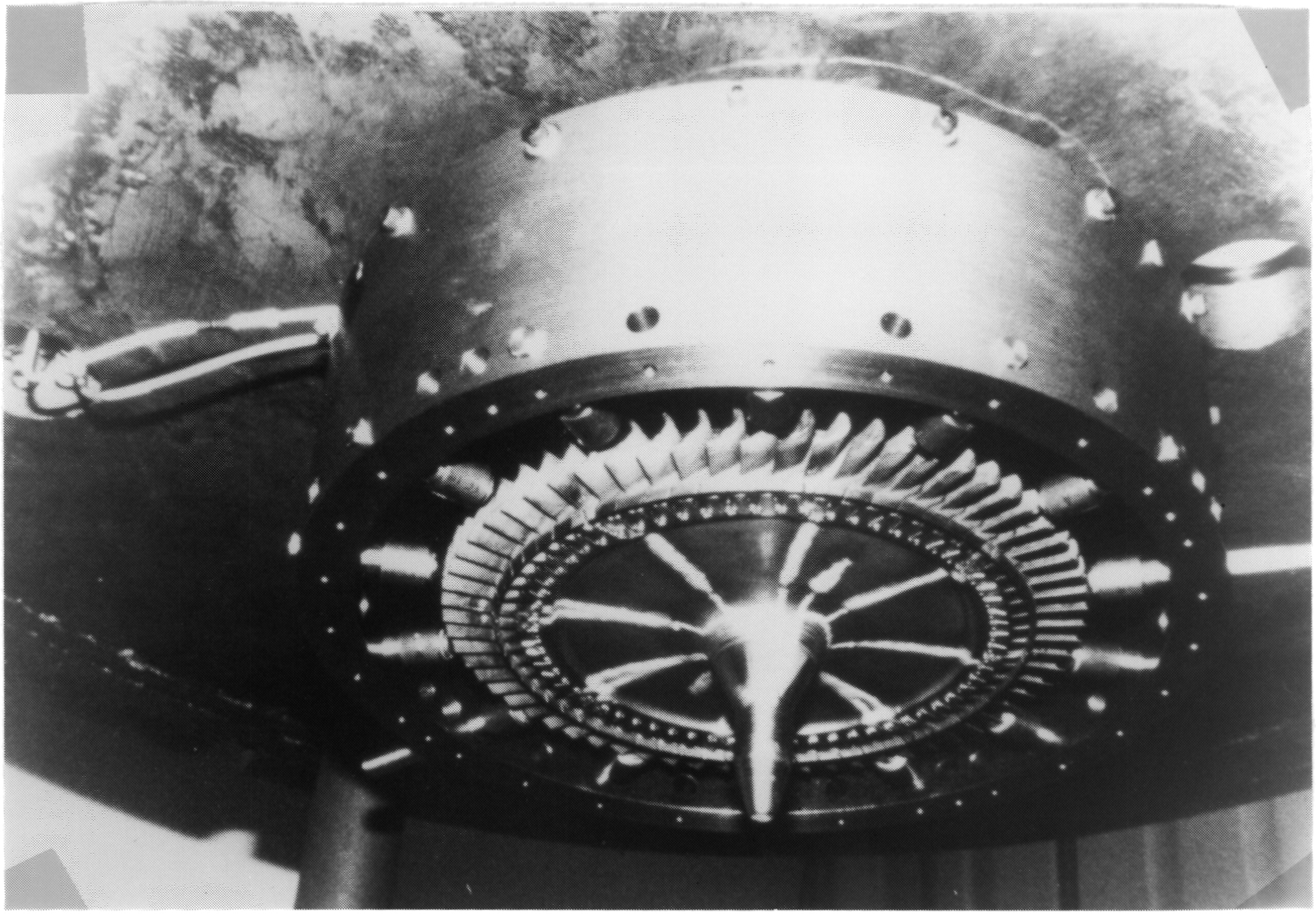


Figure 20 Second Test Disk Setup On Spin Assembly

During low speed trial spins significant one per rev deflection of the spin shaft and a high amplitude, low frequency whirl deflection of the shaft were shown by the shaft position monitors. Rebalancing of the disk solved the one per rev vibration deflections but the whirl mode deflections persisted. It was thought then that excessive friction damping of the spin shaft was occurring at the sacrificial balance disk interfaces and at the arbor shaft to quill shaft coupling. Redesign of the shaft coupling and removal of the sacrificial balance disk with subsequent rebalancing of the specimen reduced the whirl mode deflections significantly. The test series then began with one blade strain gage from each test octant and the two disk strain gages routed to the test recorder.

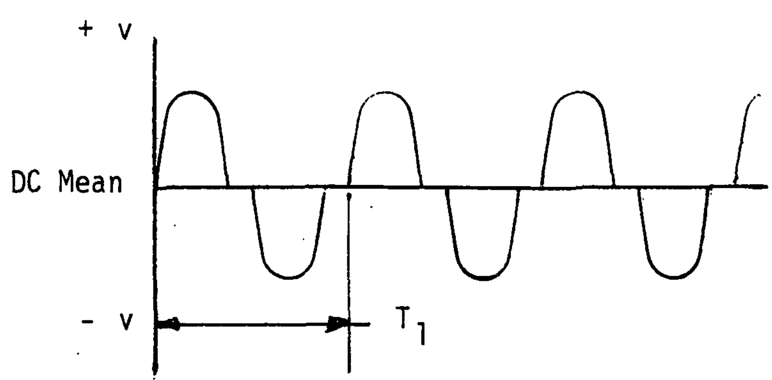
The planned spin test series included test spins with 28, 14, and if no problems occurred, with 8 radial excitation magnets installed. The 28 magnet run would be a low speed run with the 4500 Hz whole blade resonance mode being excited at about 10,000 rpm and the 8500 Hz airfoil alone resonance mode being excited at about 18,000 rpm if blade platforms were over-restrained by the dampers. It was thought this might happen for at least the heaviest dampers. The 14 magnet test was expected to induce whole blade resonance at about 20,000 rpm and airfoil alone resonance at about 36,000 rpm. The 8 magnet test would induce whole blade resonance at about 35,000 rpm. The effects of the dampers thus would be observed at several spin speeds for these two blade resonance modes.

However many magnets were installed, they were always installed with adjacent magnets having opposite polarities. Each magnet then would induce a force pulse on each blade. However, passage through the fields of two adjacent magnets would be required to induce one bipolar cycle of induced voltage. It can be shown that the frequency spectrum formed by the sum of these two signals will contain components from the two sources that are widely separated in frequency and easily identifiable.

Trial spins were made with spacing of the excitation magnets from the blade tips at 0.18, 0.15, and 0.12 inches. On April 13, 1984

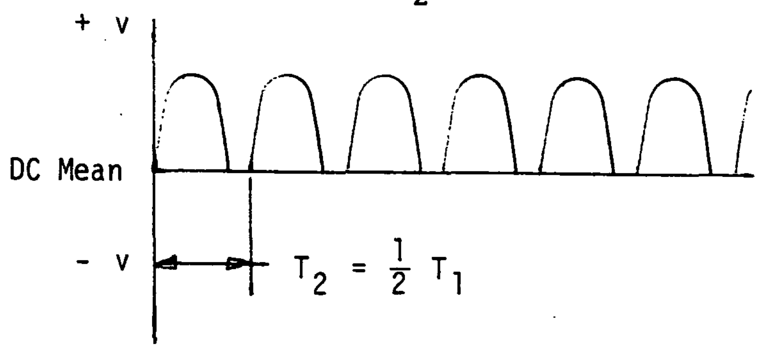
the magnet gap spacing was set at 0.10 inches and the low speed vibration run was completed. Twenty-eight excitation magnets were spaced symmetrically around the circumference of the test assembly for this test. Data was recorded over the range of 6,000 to 23,000 rpm during spin acceleration and deceleration of the disk assembly. The vibration excitation frequency range covered during this test ($f = \frac{rpm \times 28}{60}$) was 2,800 to 10,733 Hz. The spin pit absolute pressure during the test was 2.5 torr. During the test the whirl circle diameter reached 0.030 inch and the one-per-rev vibration due to unbalance reached approximately 0.004 inch P-P at 23,000 rpm, as shown by shaft deflection sensors at the shaft coupling. The whirl frequency was very low, approximately 20 Hz, or 20 whirl revolutions per second.

An explanation of the strain gage signal characteristics is required to provide a basis for understanding the data shown later in the frequency spectra of the turbine blade strain signals. The strain gage signals are the summation of several components, some of which are caused by magnetic induction due to the passage of the strain gages through the fields of the excitation magnets and some of which are due to vibratory strains in the turbine blades. For the low speed run, 28 permanent magnets were arranged symmetrically around the disk circle with alternate north and south poles facing the blade tips. Passage of the strain gages through the alternate polarity fields produces an induced pulse train of alternate electrical polarity as shown here:



The repetition frequency for this pulse train is $\frac{1}{T_1} = f_1$, where $f_1 = \frac{m}{2} \times \frac{rpm}{60}$ and m = the number of magnets. It can be shown by a

Fourier analysis that this pulse train consists of the summation of a series of sine waves with frequency components f_1, $3f_1$, $5f_1$, $(2n - 1)f_1$ for $n = 1, 2, 3, \ldots$ etc. Similarly, passage of the blade tips with the added cobalt weld beads through the magnetic fields generates flexural forces in the blades which produce a pulse train of voltage in the strain gage circuits as shown below. The repetition frequency for this pulse train is $f_2 = \frac{1}{T_2} = m \times \frac{rpm}{60}$ and $f_2 = 2f_1$.

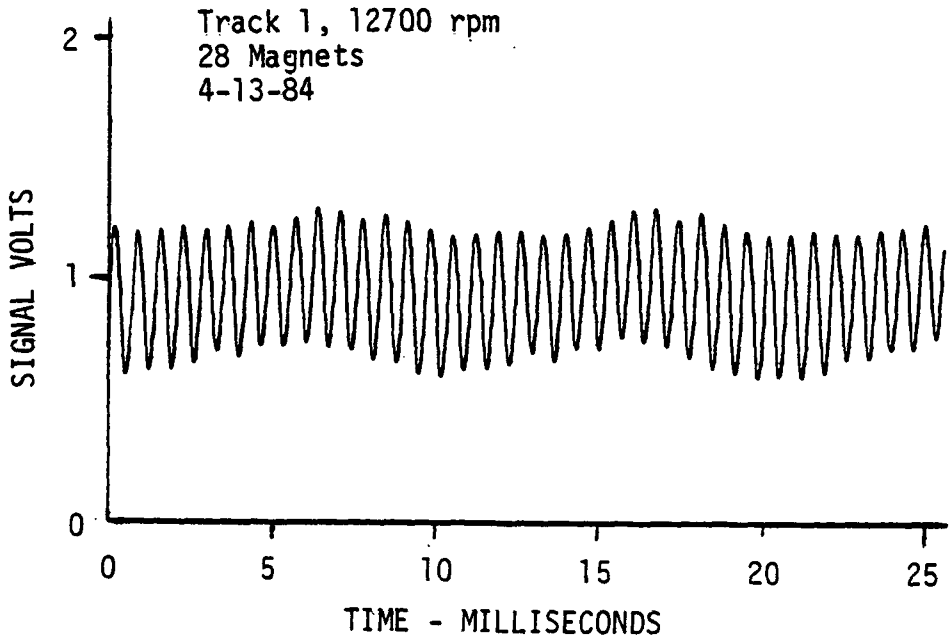


Fourier analysis shows that this pulse train is made up of a series of sine waves with frequency components f_2, $2f_2$, $3f_2$, . . . etc., and since $f_2 = 2f_1$ this series can be restated as $2f_1$, $4f_1$, $6f_1$, $2nf_1$ for $n = 1, 2, 3, \ldots$ etc. Clearly, these two sine wave series are distinct from each other. The electromagnetically induced voltage pulses will produce peaks in the strain gage signal frequency spectra at 14E (E = engine order), 42E, 70E, etc. The induced vibration pulses will produce peaks at 28E, 56E, 84E, etc. These will be the source of the major peaks in the frequency spectra. The first series is extraneous noise signals and the second is the vibration data of the blades. When the number of magnets is reduced from 28 to 14 for the high speed spin test, the first series becomes 7E, 21E, 35E, etc. and the second becomes 14E, 28E, 42E, etc.

There are additional noise sources in the data as well. The low frequency whirl of the spin shaft acts to displace the spinning disk laterally from the center of the enclosing circle of magnets. This causes the strength of the magnetic fields to vary around the spin circle at both the blade tip and strain gage locations. The result of the variation is a one-per-rev amplitude modulation signal

impressed on both the induced noise pulses and the blade vibration pulses. Further, there is a one-per-rev vibration strain pulse due to the dynamic unbalance of the disk. These one-per-rev signal components add up to a fairly strong peak in all the frequency spectra and show increasing amplitude with increasing spin speed. Since the one-per-rev signal is basically sinusoidal its components occur at 1E, 3E, 5E, etc. These low frequency components in turn form sum and difference frequency signals with the higher frequencies of the induced voltage signal sine wave series and the blade vibration strain signal sine wave series. The spinning disk assembly tends to go in and out of the whirl mode as the spin speed changes. This causes the one-per-rev signals due to whirl displacement to vary and they may be strong or weak at a particular time, dependent on the current whirl condition. The one-per-rev signal due to dynamic unbalance increases monotonically as the second power of the spin speed. All of the dynamic signals ride on an increasing DC signal level which represents the strain at the gage location caused by the centrifugal force induced on the blade by the current spin velocity.

Time series signals from an undamped turbine blade strain gage are shown in Figure 21. The upper tracing is taken from a spin at 12,700 rpm with 28 excitation magnets. The lower tracing is taken from a spin at 25,400 rpm with 14 excitation magnets. The fundamental frequency of both signals is 2963 Hz, the induced voltage pulse repetition frequency of 14E for the upper trace and 7E for the lower trace. The one-per-rev whirl effects can be seen as amplitude modulation of the fundamental signal. The one-per-rev forced vibration signal due to unbalance appears as the variation in the mean of the fundamental. The forced vibrations due to magnetic force pulses at 28E and 14E respectively are very small at this off-resonance condition. The frequency spectra derived from these two time signals by an FFT analyzer are shown in Figure 22. The major frequency peaks occur at 1E, 14E, and 42E for the low speed spin and at 1E, 7E, and 21E for the high speed spin. The signal levels between these major peaks are due to the mixing of these peaks and their harmonics with each other. The effects of the larger amplitude of the one-per-rev signal and its



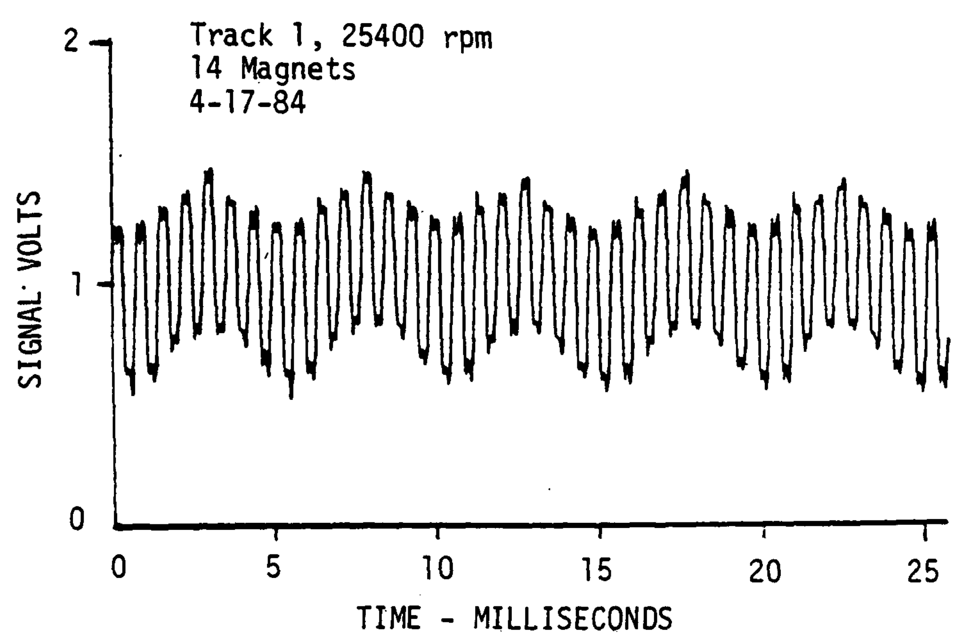
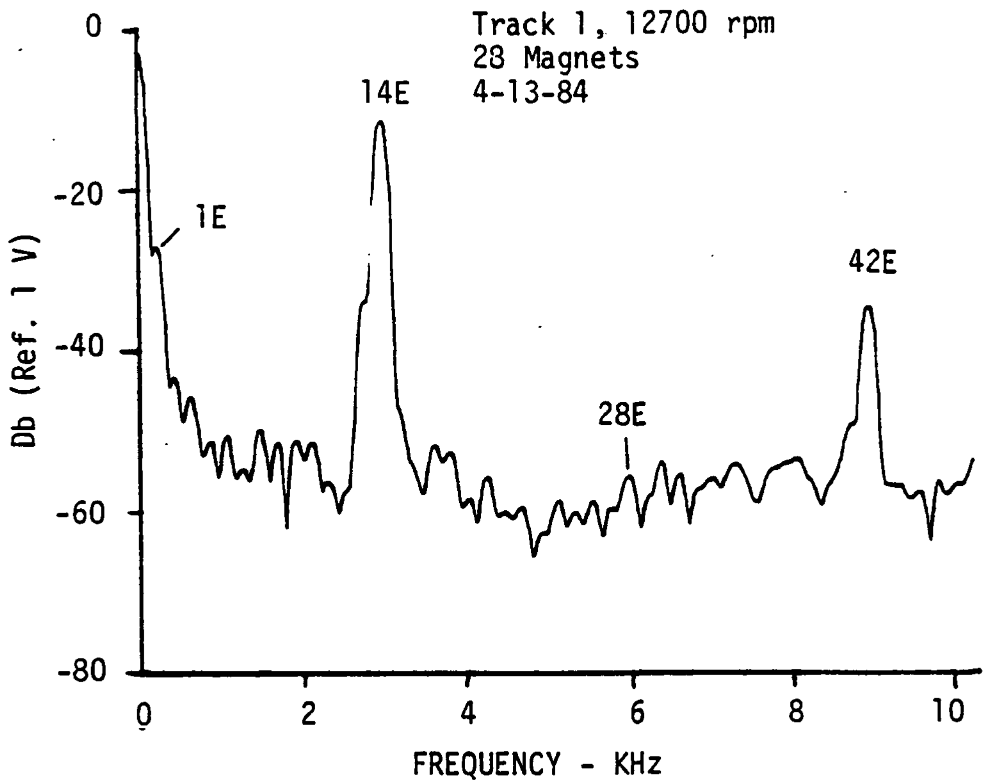


Figure 21 Track 1 Signal Time Segments- No Resonance



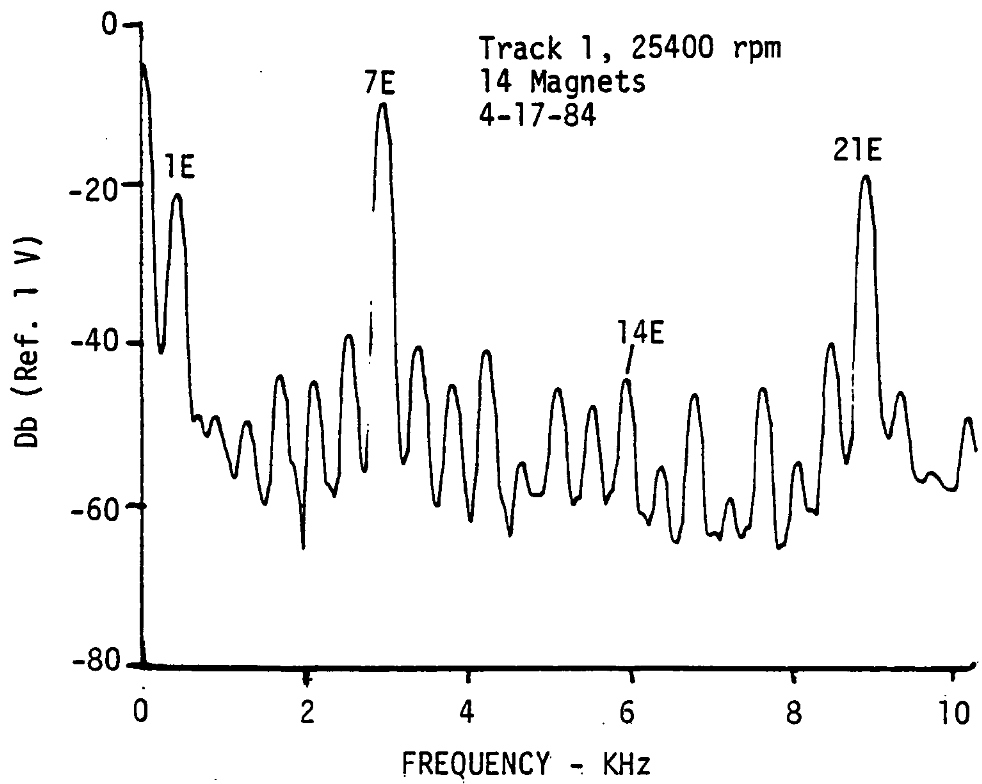


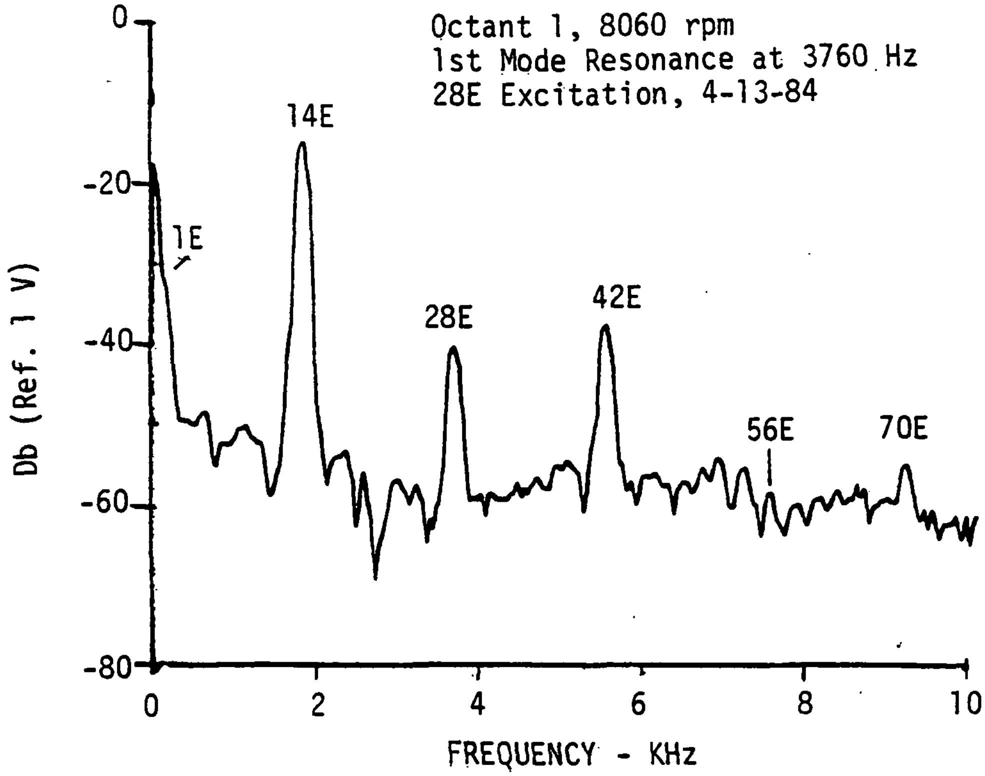
Figure 22 Track 1 Frequency Spectra- No Resonance

harominics are seen clearly in the spectrum from the high speed spin data. The signals shown in these two spectra consist almost entirely of noise voltage signals with respect to the blade resonance vibration signals that are the real test data. With this information in mind we can proceed to discussion of the test data.

During the low speed spin test of April 13, 1984 all eight of the monitored turbine blades showed first bending mode resonant vibration during the spin acceleration in the frequency range of 3760 to 4880 Hz, corresponding to a spin speed range of 8,000 to 10,500 rpm with the 28E magnetic excitation pulses. The frequency spectra of the turbine blade strain gage signals during these resonance vibrations are shown in Figures 23 through 26. Figure 23 shows the data from the two blades with no friction dampers. Figure 24 is the data from blades with 0.56 gram production friction dampers. Figure 25 shows the data from blades with 0.10 gram experimental dampers. Figure 26 shows the data from blades with 0.20 gram experimental dampers. The two spectra shown in each of these figures are for strain-gaged blades located diametrically across the disk in the middle of test octants of eight adjacent similarly configured blade-damper installations. The first mode resonance frequencies of all these blades are surprisingly lower than expected. The first mode resonance frequencies for these blades from the bench tests when they were hard-clamped in a broach block by a locking bolt were as follows:

Octant	fr_1-Hz	Octant	fr_1-Hz
1	4950	5	4725
2	4850	6	4850
3	4900	7	4950
4	4725	8	4775

It seems obvious that these blades were not yet hard-clamped in the disk firtree slots when the first mode resonance occurred during the spin acceleration. Otherwise the blades with dampers would be expected to resonate at frequencies higher than those of the bench test, as



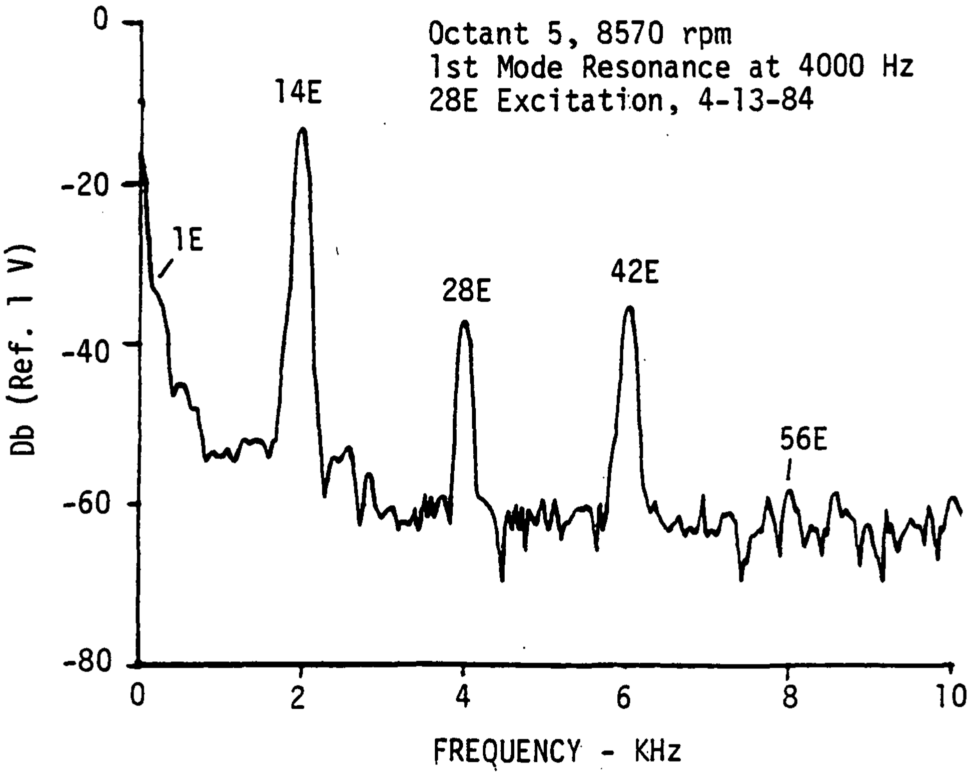
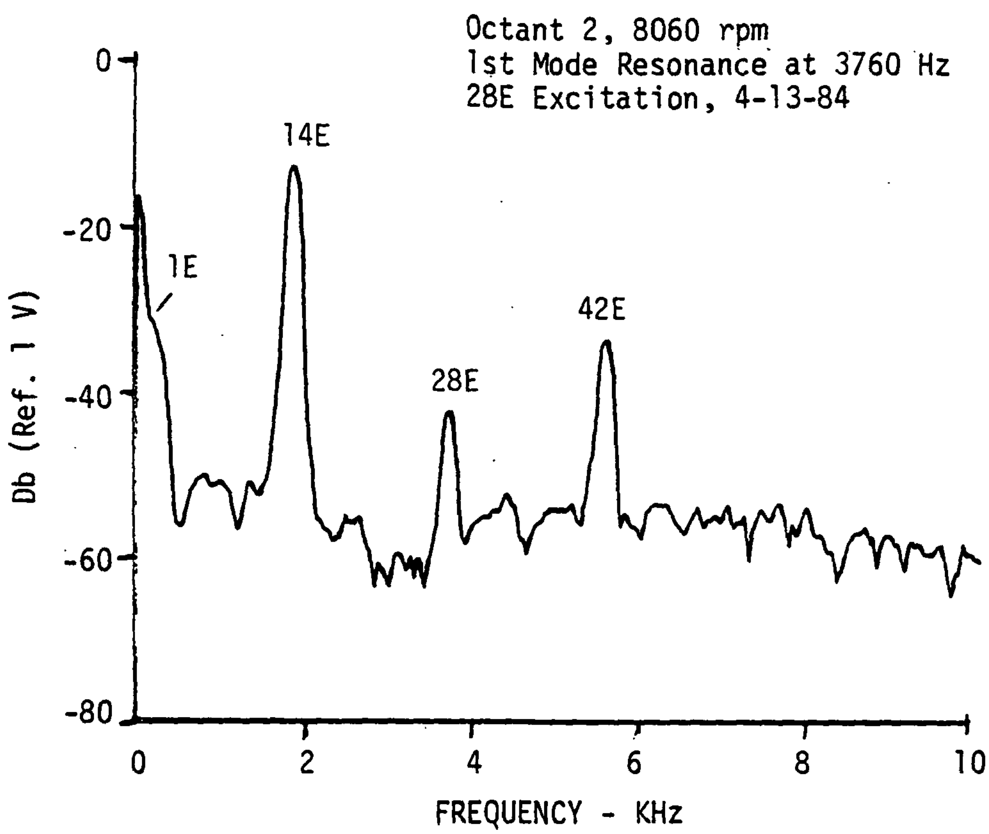


Figure 23 Vibration Spectra - Undamped Blades



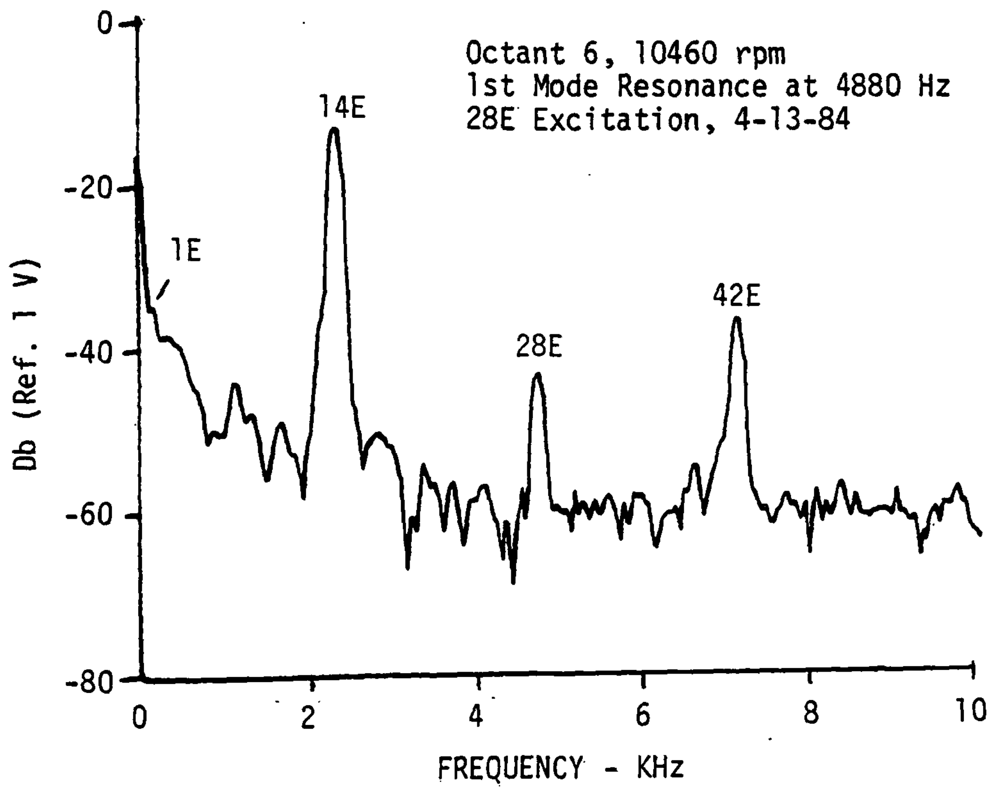
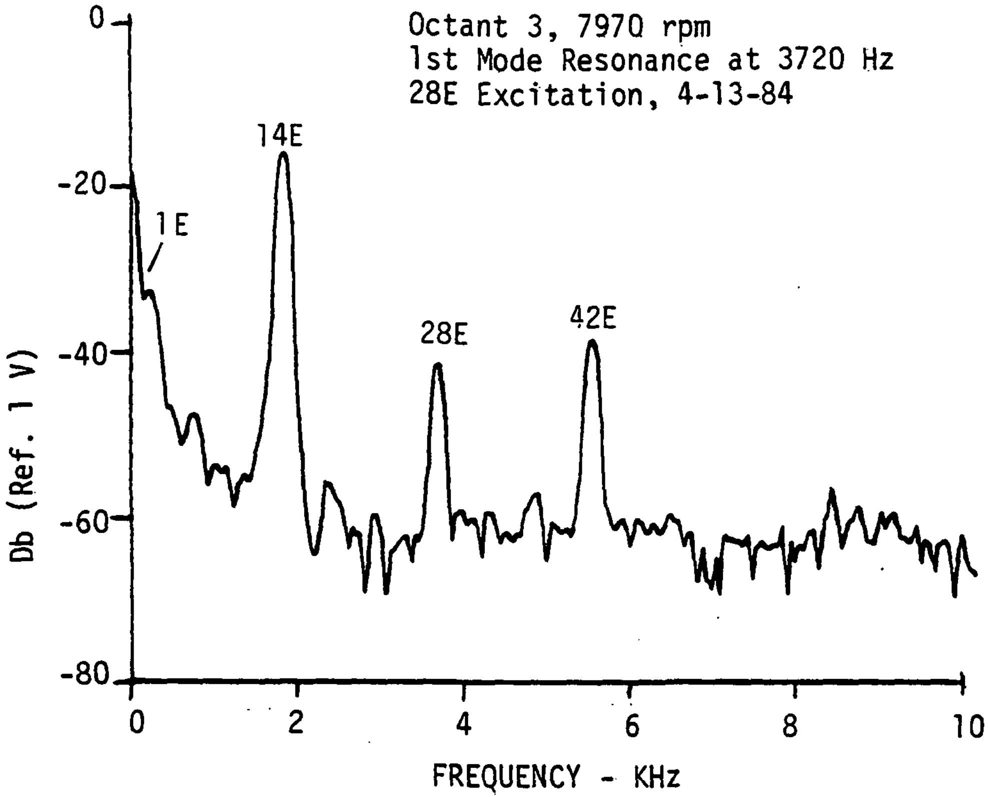


Figure 24 Vibration Spectra - Production 0.56 gram Dampers



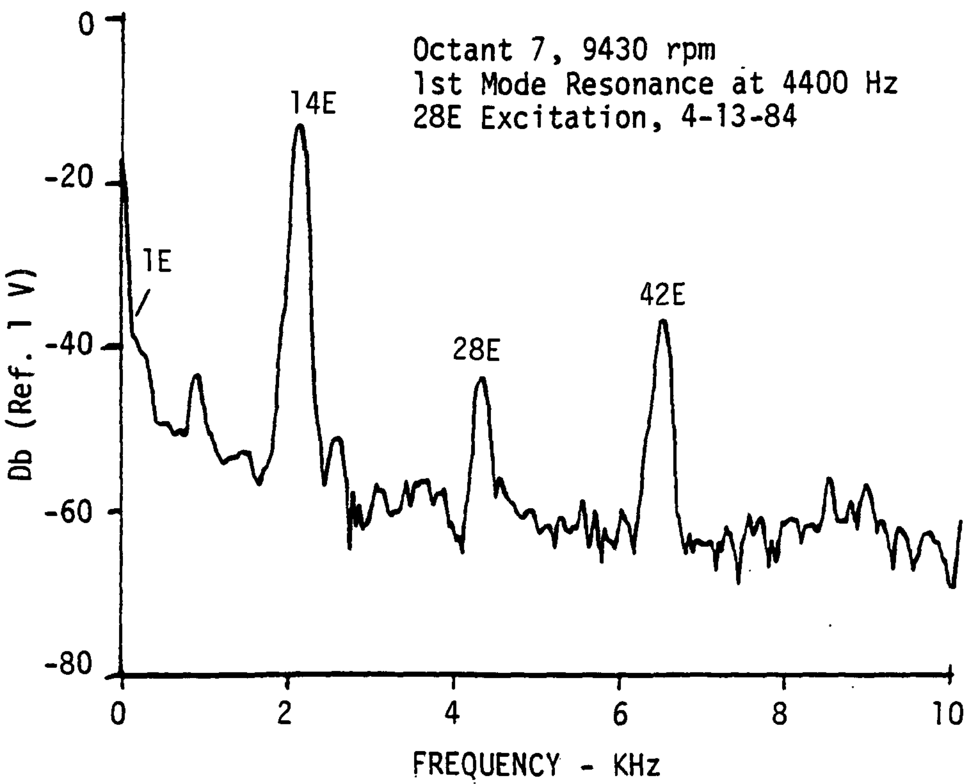
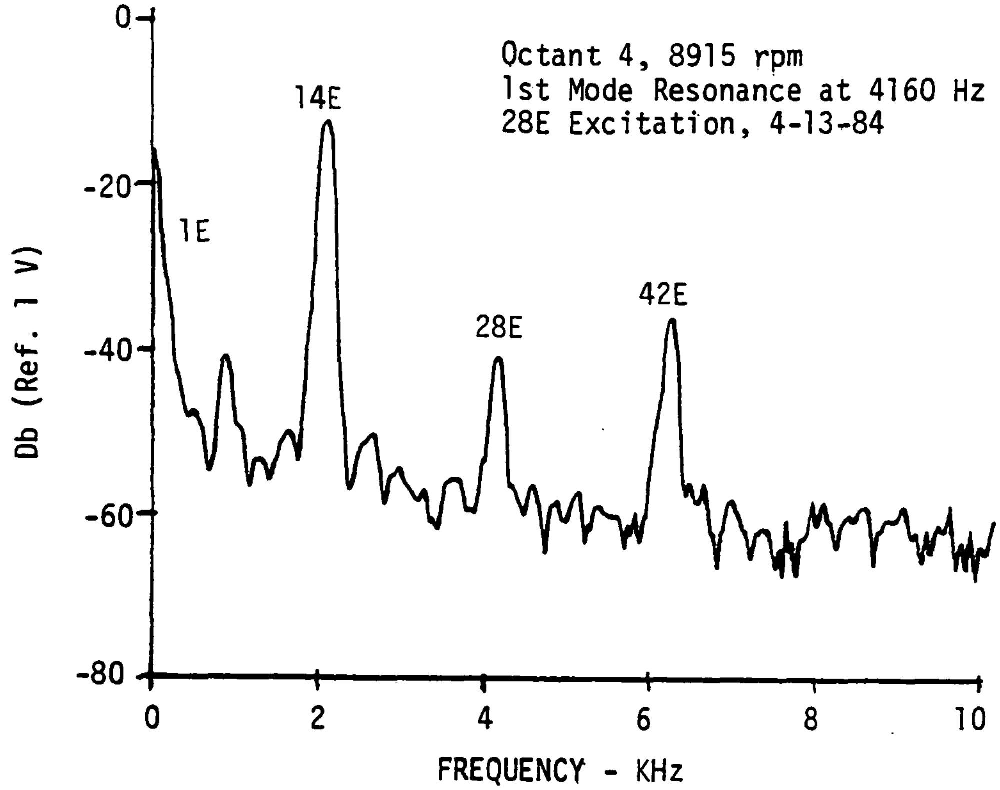


Figure 25 Vibration Spectra - Experimental 0.10 gram Dampers



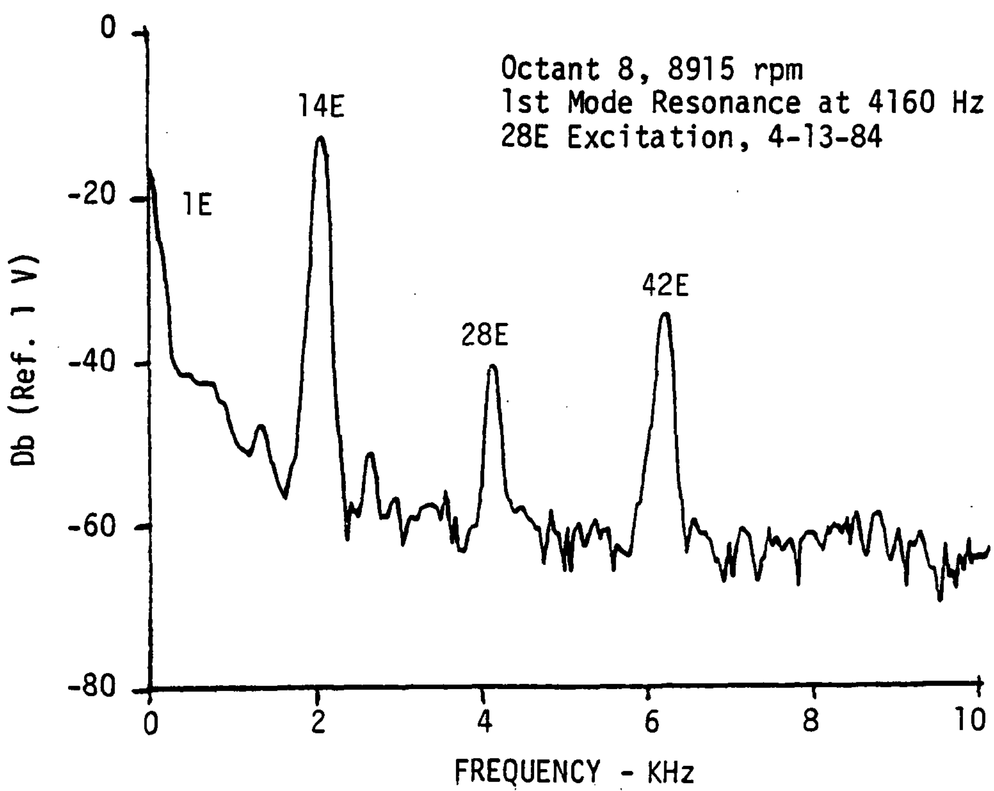
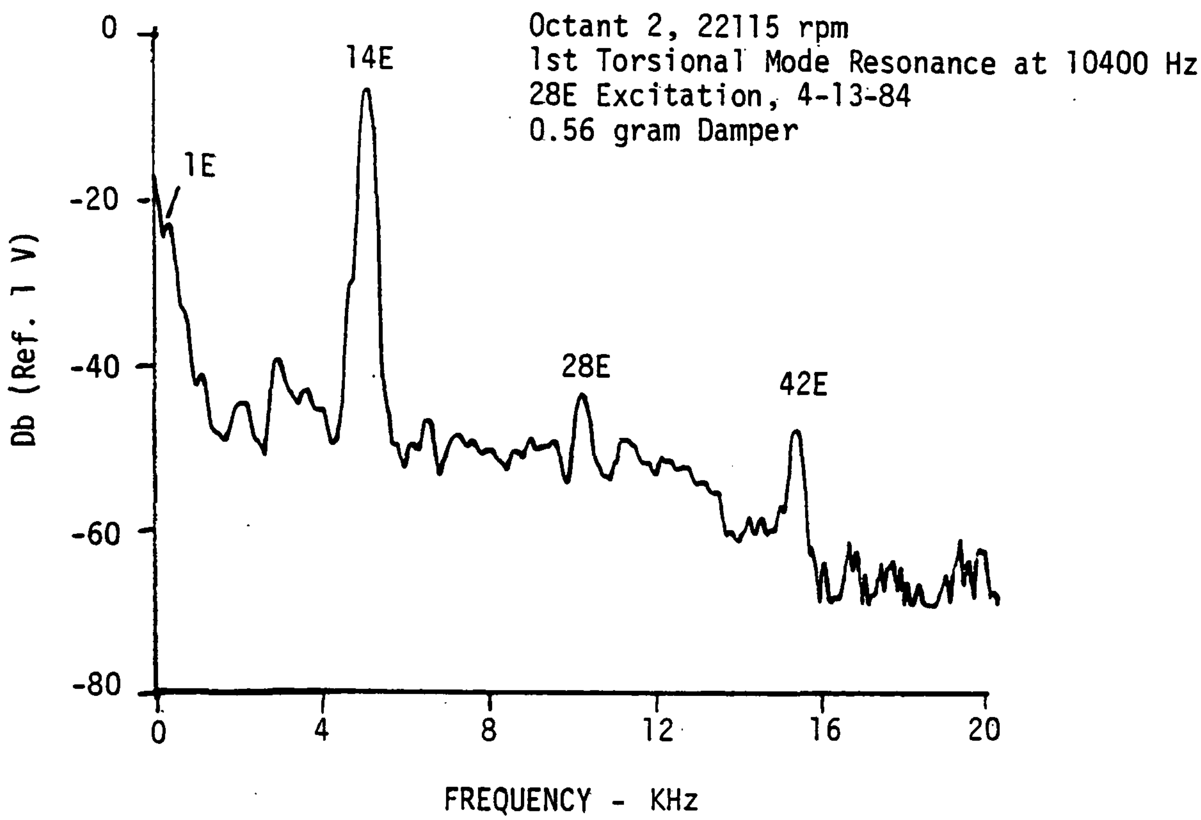


Figure 26 Vibration Spectra - Experimental 0.20 gram Dampers

shown later by the analytical study. The loose (relatively) firtree fit reduced the stiffness of the blade-socket system, reducing the resonance frequency. This being the case, the higher resonance frequency shown by the blade in octant 6, which has a production damper, may be indicative of friction damping. If so, it is not known why a similar amount of friction damping did not occur for the similar blade-damper configuration in octant 2.

The instrumented blade in octant 6 was the last blade to resonate in the first bending mode during the spin acceleration, and that resonance occurred at 10,460 rpm. It was expected that at least the blades with 0.56 gram production dampers might experience damper lockup at approximately 18,000 rpm and resonate at 8500 Hz in the airfoil alone bending mode. This could occur because the magnetic excitation force remains constant with rpm while the damper normal force increases as the square of the rotational velocity. That airfoil alone resonance mode did not occur on any of the instrumented blades during the low speed spin test. The only other vibration phenomena that occurred during the spin acceleration from 10,500 to 23,000 rpm was that the blades in octants 2 and 3 were excited at a fairly low amplitude in the first torsional mode resonance, as shown in Figure 27. The other six instrumented blades must experience first torsional mode resonance at frequencies higher than 10,750 Hz, the excitation upper frequency limit reached during the low speed spin test.

After completion of the low speed spin test on April 13, 1984, one-half of the 28 magnets were removed and the remaining 14 magnets were arranged symmetrically around the disk in the alternate polarity configuration for adjacent magnets. The high speed spin test then was attempted on April 17, 1984. Spin acceleration to 31,000 rpm was achieved in this test. At that spin speed an excessive whirl mode deflection occurred followed immediately by failure of most of the strain gage circuits. Examination of the spin structure showed that most of the epoxy cement used to hold the strain gage circuit leads to the lower arbor had unbonded. The subsequent vibration



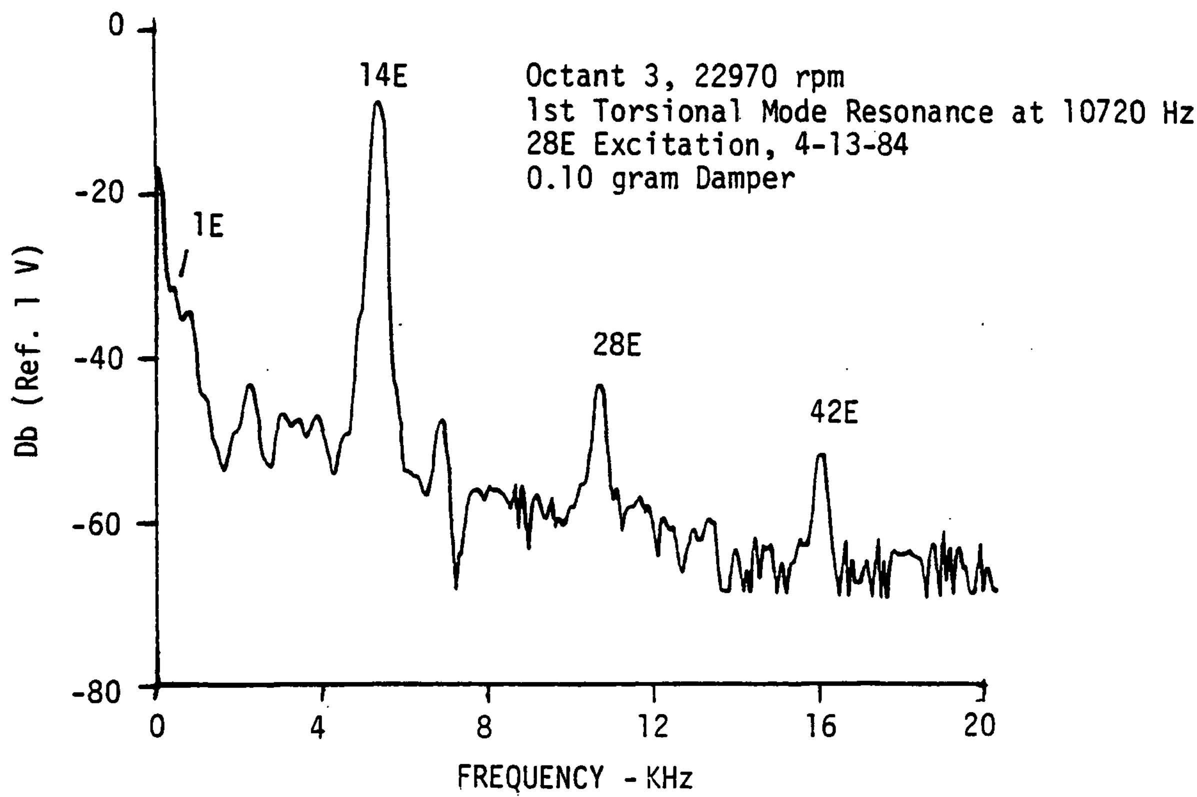


Figure 27 Vibration Spectra at 1st Torsional Mode Resonances

during braking of the assembly had caused the screws holding the slip ring assembly to the drive turbine housing to back out. This allowed the slip ring case to rotate, breaking the strain gage circuit leads. At that point the decision was made to rewire the strain gage circuits from the disk (using a different epoxy) and to replace the slip ring set with a new assembly, using lockwashers on the attachment bolts. The test disk-arbor assembly required rebalancing after installation of the new strain gage circuit leads.

Examination of the data from this high speed spin with 14E excitation showed first mode flexural resonance vibration only on the undamped blades in disk octants 1 and 5. Frequency spectra for the resonance condition on those two blades are shown in Figure 28. The resonances occurred at slightly higher frequencies than in the low speed spin test, 4000 Hz compared to 3760 Hz for the blade in Octant 1 and 4240 Hz compared to 4000 Hz for the blade in octant 5. The increases in these resonance frequencies are attributed to the four-fold increase in centrifugal force on the blades that was reacted at the firtree slots.

The fact that no resonance vibrations were detected during the spin acceleration to 31,000 rpm on the six blades with platform dampers is indicative that the dampers were working effectively to limit the first mode resonance vibration.

The repaired and rebalanced test assembly was installed in the spin pit with a new slipring assembly on May 4, 1984. On May 7, 1984, another high speed spin was attempted with 14 excitation magnets. That run resulted in failure of the slip ring because blockage occurred in the coolant ports and overheating of the contacts occurred at 30,000 rpm. That slip ring assembly then was replaced by ASI so that spin testing could be continued.

On May 14, 1984 a high speed spin test was completed to 38,300 rpm but the data set for the test was not completely satisfactory. Noise began occurring on several of the strain gage channels and no useful data was recorded after 33,000 rpm was reached. Again, first bending mode vibration occurred on the undamped blades near 4200 Hz and 18,000 rpm.

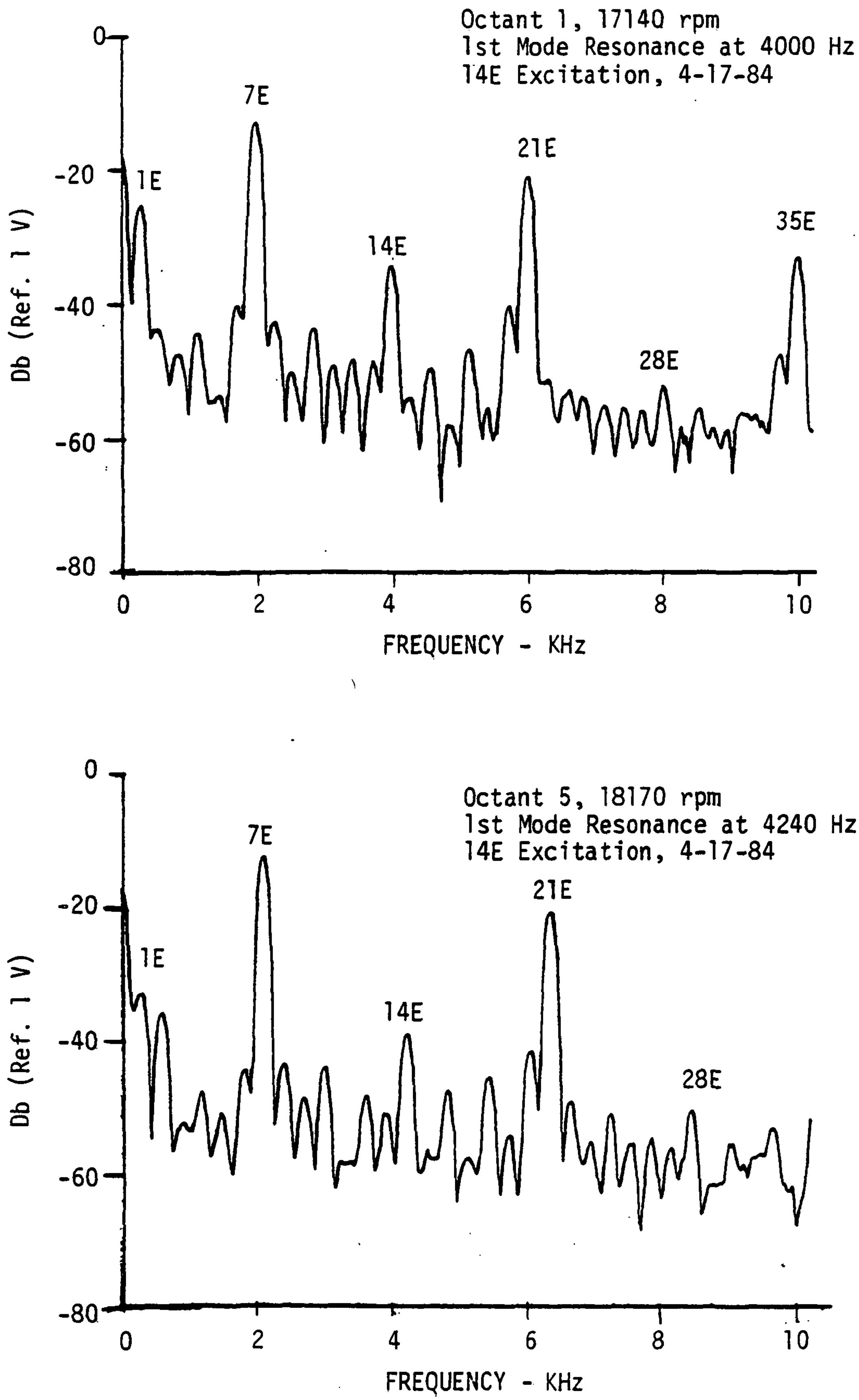
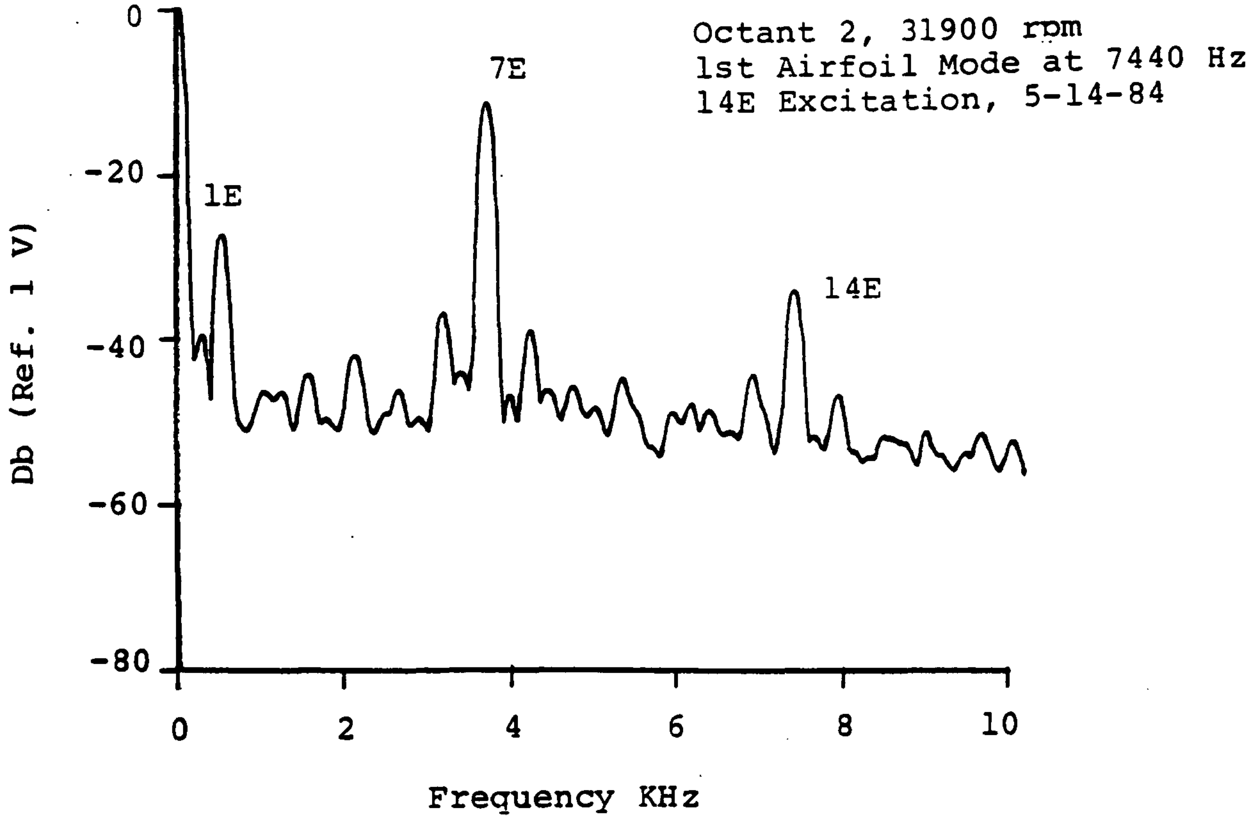


Figure 28 Vibration Spectra - 14E Excitation of Undamped Blades

The airfoil alone bending mode vibration occurred on the two blades with production 0.56 gram friction dampers at 7500 Hz just prior to the loss of signal from those blades at 33,000 rpm. Spectral plots of the strain gage signal at resonance for those two blades is shown in Figure 29. The reduction in resonance frequency from the expected value of 8500 Hz to 7500 Hz occurred in this case not because of the amount of fixity at the firtree root but because of the addiiton of the magnetically permeable mass to the tip of the airfoil section. We believe even higher values of these airfoil alone flexural resonance modes was not shown because the strain gage circuits on those two blades deteriorated to noise immediately after the data shown in Figure 29 was recorded. It is significant that the value of the resonance peak shown for the octant 6 blade is approximately 6 db higher than any of the resonance peaks recorded for undamped blades in octants 1 and 5 during any of the test runs. That amplitude ratio indicates a dynamic stress ratio of at least 2:1 at the measurement location for an overdamped blade as compared to an undamped blade. We feel that this ratio would have increased if we could have recorded data at a slightly higher spin speed because the resonance response amplitude was still increasing when the data signal was lost. The high response amplitude for the damped blade indicates overdamping causing platform fixity, and confirms the large airfoil alone flexural response shown in the analytical study.

After the high speed spin run on May 14, 1984, three additional attempts were made to acquire high speed blade vibration data on May 16, 18 and 22, 1984. Each of these runs experienced noise on the strain gage circuits beginning at spin speeds as low as 20,000 rpm despite the fact that slip ring rotors were changed and brush tension was adjusted after each run. It was determined that a mixture of open circuits and grounds were occurring in the strain gage circuits at high speed spin that could not be detected under static conditions, evidently due to fatigue of the gage circuits. The test program was terminated at that time because funds were not available to perform the extensive disk and blade rewiring that would have been required to obtain additional high speed spin test data.



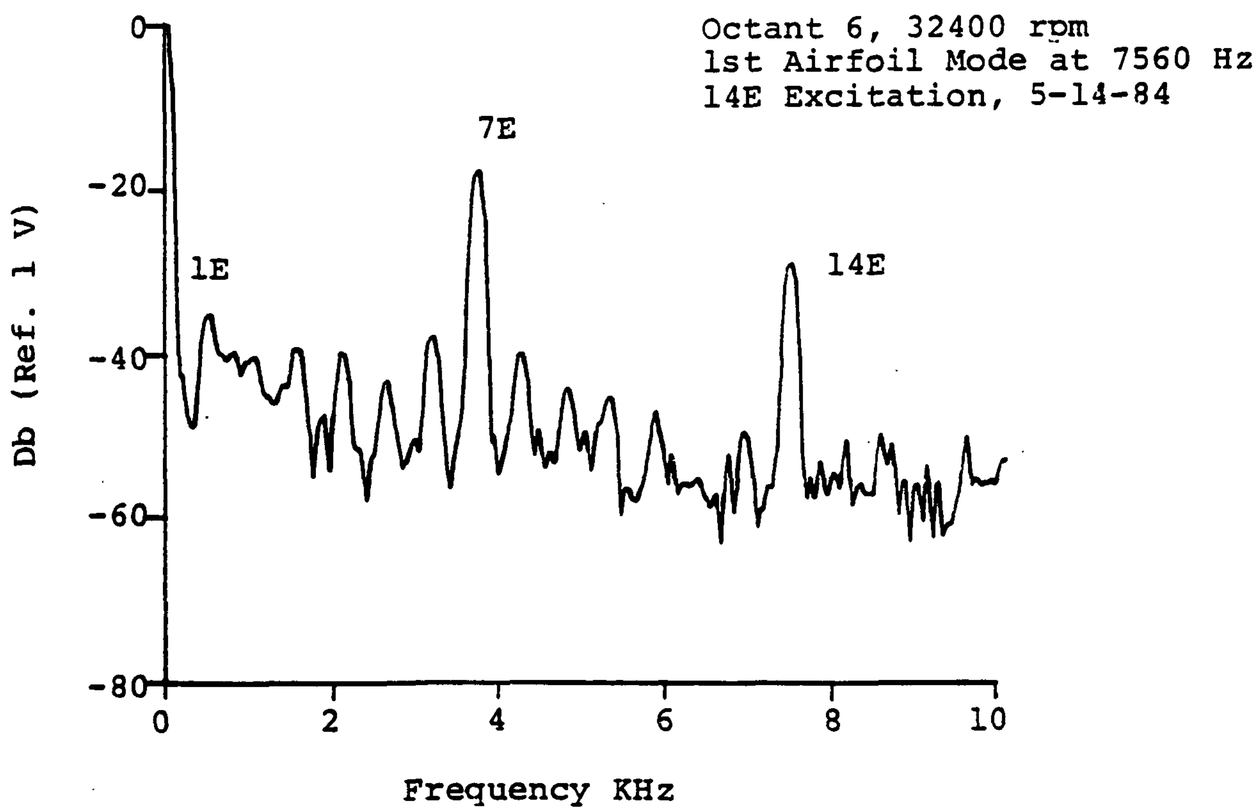


Figure 29 Vibration Spectra - Airfoil Flex Mode - Production 0.56 Gram Dampers

However, this spin test series did show the low frequency whole blade bending mode for undamped blades, the airfoil alone bending mode for blades with the heaviest dampers, and the absence of response in either of these modes for blades with light weight trial dampers. Further, these responses were induced at the 14E excitation frequency judged to be the major component of the flow pulse pattern induced by the HPFTP front bearing support struts. The spin tests also showed that the root fixity at the firtree was much lower than expected for the HPFTP blades for spins at least as high as 18,000 rpm. It would be expected that relatively high root damping values would accompany low root fixity at lower spin speed values and that damping values would decrease appreciably as root fixity increased with increasing spin speed.

4.2 ANALYTICAL EFFORT

The analytical effort proposed for use in the program was the lumped parameter method, often called the lumped mass method, developed by Jones and Muszynska over the last several years. A computer program utilizing this method was in existence and UDRI had adapted this program for implementation on the VAX 11/780 computer. The computer program was being used in a single blade mode to predict the effects on the blade vibration response of blade platform friction dampers operating between the blade platform and the disk. The first runs of the computer program in the multiblade mode with blade to blade platform friction damping did not seem to produce enough blade tip vibration deflection to explain the early HPFTP test run failures and the continued high cycle fatigue problem causing fatigue cracks in the blade airfoils just above the blade platforms. UDRI then proposed to develop a simplified finite element analysis (FEA) computer program to predict the vibration response of the friction damped blade.

The FEA program was developed and is documented as UDR-TM-82-08, included as Appendix A to this report. When that program was exercised it produced almost exactly the same results as the lumped mass analysis computer program but at a ten times greater computer run time cost because the FEA program had to proceed through a transient analysis

run to arrive at the steady state solution. It was decided then to work toward improvement of the lumped mass analysis as that seemed the most cost effective way to proceed with the analytical study.

During the course of the study, several changes were made in the lumped mass analysis. The most important computer program change was the introduction of individual loss factor values for the inboard and outboard sections of the lumped parameter model of the blade. However, the most significant changes consisted of new viewpoints for looking at the output data sets from the analysis computer runs. These changes included the considerations: first, of the blade airfoil deflection with respect to the blade platform deflection rather than with respect to the fixed blade root as being more reprsentative of the failure mode of the blade; and second, that stick-slip or stick friction conditions of the dampers with the blade platforms would prove even more damaging to the blades and would represent the conditions which caused the HPFTP test stand failures.

The descriptions of the lumped parameter analysis and of the parametric study performed using that analysis are included in the remainder of this report section. The parametric study considers the effects on the blade deflection response of variations in: the coefficient of friction; the normal force on the friction surface interface; the blade hysteretic damping; the blade to blade phase angle of the harmonic forcing function; and the amplitude of the forcing function. The results of the study are applicable to any blade-damper system similar to the one used in the study.

The end product computer program and its use are documented in UDR-TR-84-38, the program user's manual, which is included as Appendix B of this report. The users manual and a digital tape containing the program were delivered to NASA/MSFC on June 27, 1984.

4.2.1 The Lumped Parameter (Lumped Mass) Analysis

The lumped mass analysis evaluates a blade only in its lower order flexural modes and only for the steady state solution. The blade is represented by two concentrated masses (m_1, m_2) supported

in series by two flexural springs (k_1, k_2) with a hysteretic loss factor (η) associated with the springs. The hysteretic loss factor represents the combination of root damping, aerodynamic damping, and material damping in the operating blade. The concentrated masses and flexural springs represent the modal parameters of the blade in the flexural plane. The modal parameters for the SSME HPFTP first stage blade can be determined from the resonance equations of the blade in three flexural resonance conditions, as shown in Figure 30. The resonance frequencies f_1, f_2, f_3 have been measured in test programs as follows: f_1 has been measured by Rocketdyne Division of Rockwell International (RDRI) in siren tests and by UDRI in impact tests; f_2 has been measured by RDRI in siren tests; f_3 was shown in the RDRI whirligig test data on blades with welded platforms and on blades with friction dampers when the dampers greatly limited platform motion. These values are average values for several blades.

Three resonance equations can be derived, two for the free blade and one for the platform locked blade, as below.

$$\frac{k_1 k_2}{m_1 m_2} = (4\pi^2 f_1 f_2)^2 \tag{1}$$

$$\frac{k_1+k_2}{m_2} + \frac{k_1}{m_1} = 4\pi^2 (f_1^{\,2}+f_2^{\,2}) \tag{2}$$

$$\frac{k_1}{m_1} = (2\pi f_3)^2 \tag{3}$$

These three equations can be simplified algebraically to state any three of the unknown parameters in terms of the fourth, such as:

$$k_1 = (2\pi f_3)^2 \, m_1 \tag{4}$$

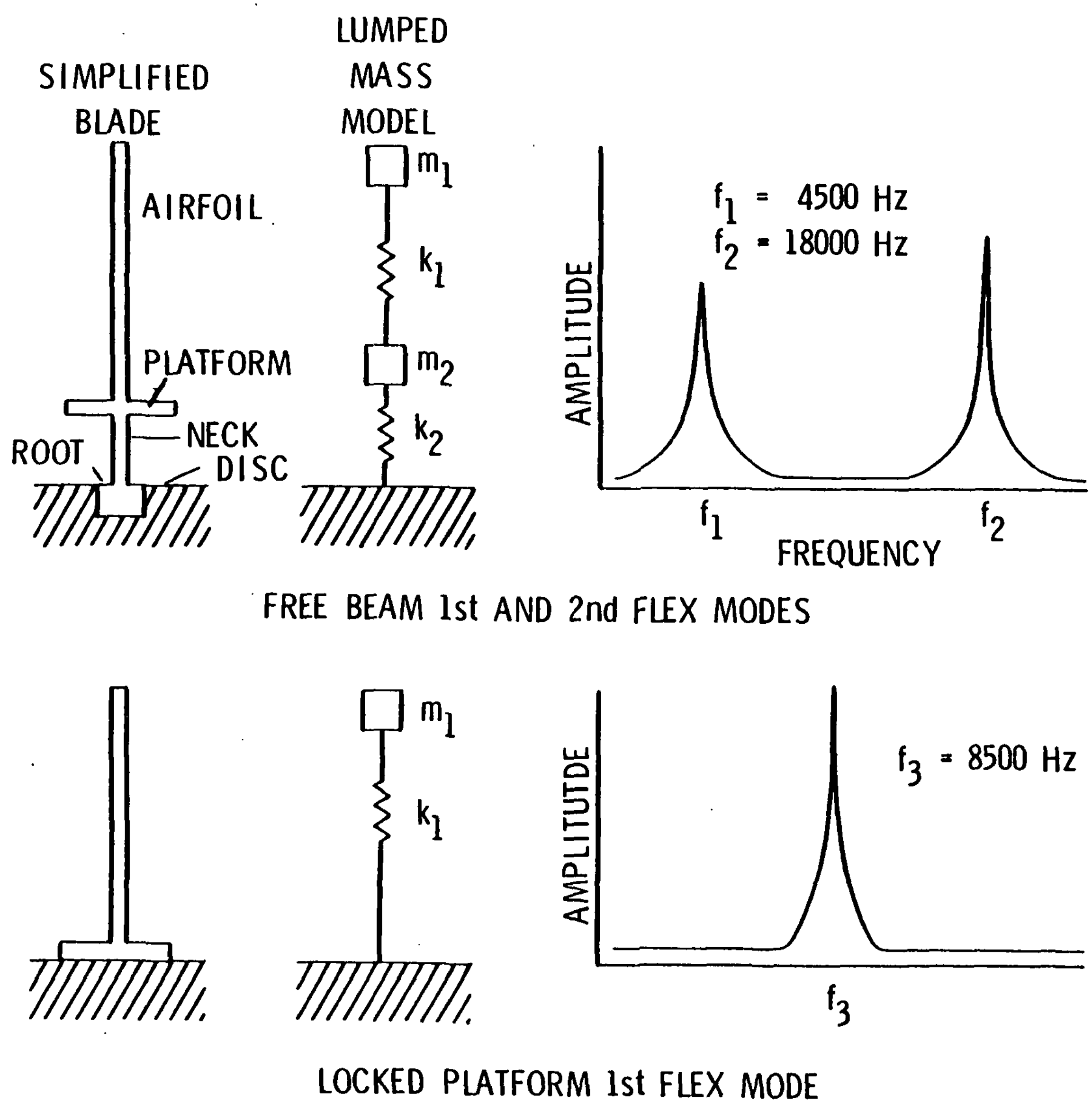


Figure 30 HPFTP 1st Stage Blade Flex Modes

$$k_2 = \frac{(2\pi f_1 f_2 f_3)^2}{(f_2^2 - f_3^2)(f_3^2 - f_1^2)} \quad m_1 \tag{5}$$

$$m_2 = \frac{f_3}{(f_2^2 - f_3^2)(f_3^2 - f_1^2)} \quad m_1 \tag{6}$$

Then if m_1 is assigned a value the other three parameters are defined. Any consistent set of units can be used. We assigned m_1 = 0.02 pound, then m_2 = 0.007975 pound, $k_1 = 5.705\text{x}10^7$ pounds/inch, and $k_2 = 2.859\text{x}.10^7$ pounds/inch.

In early analyses the same hysteretic loss factor (η) was used with both modal springs (k_1, k_2) of the blade. Intuition suggests that root damping should not be included in the hysteretic loss factor for the k_1 spring. Instead, a separate loss factor should be used with a value much less than the k_2 loss factor because of non-availability of root damping to control the dynamic response of the airfoil section of the blade. Because of this the discrete blade model was altered to include a loss factor η_1 with spring k_1 and a loss factor η_2 with spring k_2.

If a series of these blades are installed in a rigid disk with platform friction dampers between the blades and with airfoil excitation forces imposed, the discrete bladed disk model shown in Figure 31 is evolved. This system is a modal analog of the HPFTP first stage bladed disk in the frequency range of 0 to perhaps 20,000 Hz.

The equations of motion for the νth blade in Figure 31 are:

$$m_{1\nu}\ddot{x}_{1\nu} + k_{1\nu}(x_{1\nu} - x_{2\nu}) + \frac{k_{1\nu}\eta_1}{\omega}(\dot{x}_{1\nu} - \dot{x}_{2\nu}) = S_\nu \cos(\omega t + \delta_\nu) \tag{7}$$

$$m_{2\nu}\ddot{x}_{2\nu} - k_{1\nu}x_{1\nu} + \frac{k_{1\nu}\eta_1}{\omega}(\dot{x}_{2\nu} - \dot{x}_{1\nu}) + \frac{k_{2\nu}\eta_2}{\omega}\dot{x}_{2\nu} + \mu N_\nu \text{sign}(\dot{x}_{2\nu} - \dot{x}_{2,\nu+1})$$

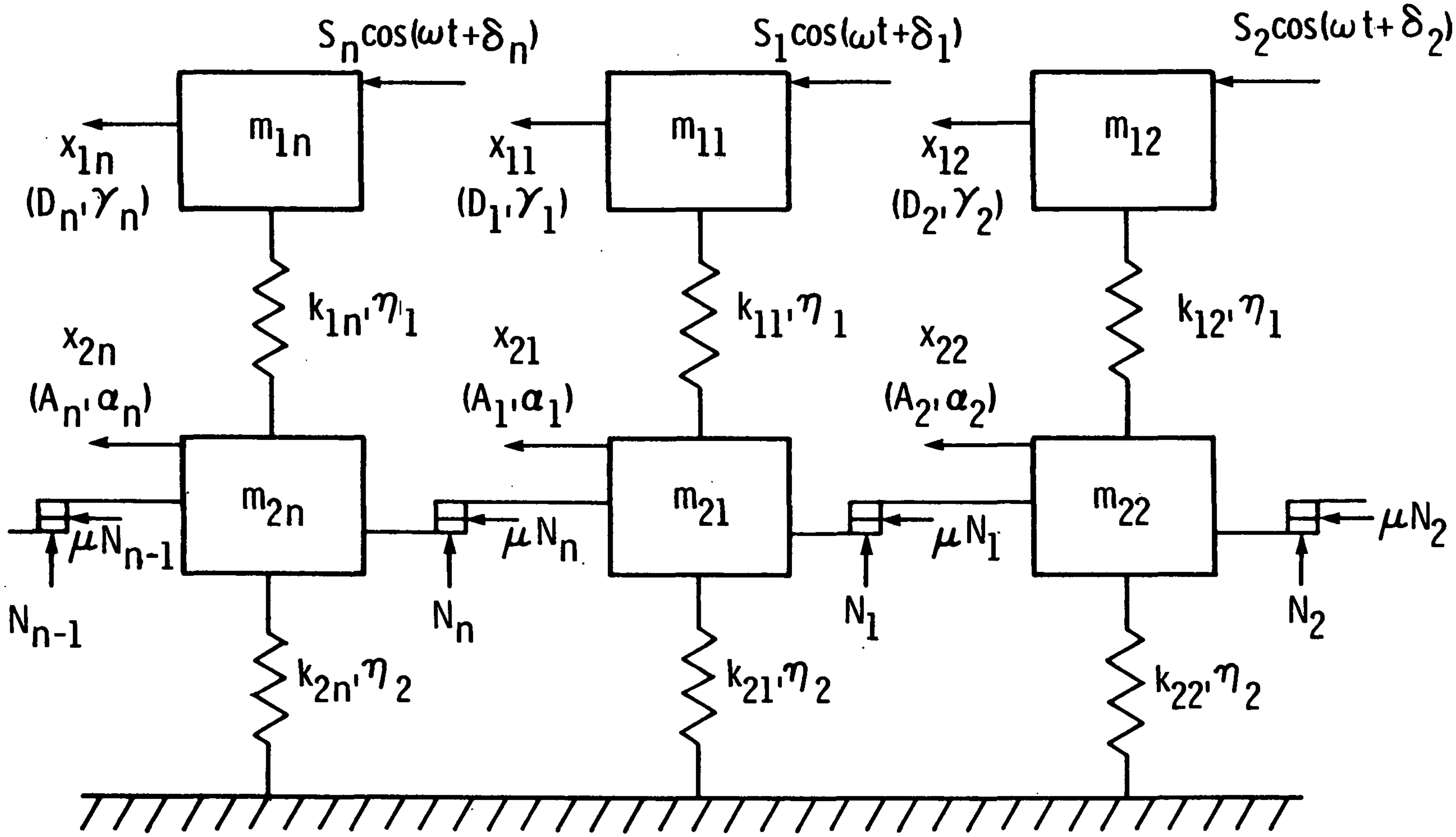


Figure 31 Lumped Mass Model of Bladed Disk System

$$+ \mu N_{\nu-1} \operatorname{sign}(\dot{x}_{2\nu}-\dot{x}_{2,\nu-1}) + (k_{1\nu}+k_{2\nu})x_{2\nu} = 0 \tag{8}$$

for $\nu = 1, 2, \ldots, n$, where n is the number of blades in the system. These are a set of nonlinear equations of the second order, the only nonlinear terms representing a Coulomb model of the friction forces on the platform. The previously undefined terms in these equations are the harmonic excitation force, $S_\nu \cos(\omega t+\delta_\nu)$, the coefficient of friction, μ, the normal force on the friction surface, N_ν, and the deflections of the modal masses, x_1 and x_2. The phase angle, δ_ν, represents the time lag of a traveling wave excitation around the disk system. This is representative of the spinning blade system passing through pressure disturbances caused by the nozzle vane and shaft front support strut wakes.

This system of equations is solved using the method of harmonic balance. A nonlinear matrix iteration is used to obtain a numerical solution. The solution, obtained by computer, consists of the deflection amplitudes D_ν, A_ν, and the phase angles γ_ν,α_ν, for the outboard (airfoil) and inboard (platform and neck) modal masses of each blade as a function of the system parameters $m_{1\nu}$, $m_{2\nu}$, $k_{1\nu}$, $k_{2\nu}$, N_ν, S_ν, δ_ν, μ, η_1, η_2, and ω. Since most of the system parameters are input to the computer as arrays, i.e., all the ν-subscripted parameters, many types of mistuned systems can be evaluated. For a tuned system the arrays are filled with identical values except for the δ_ν array. Only solutions for a tuned system have been evaluated to date, specifically, a tuned system with the HPFTP average blade. To vary N, μ, or η a series of runs must be made. Analyses have been made with $\eta_2 = 0.008$, 0.005, or 0.002, $\eta_1 = 0.1\ \eta_2$, $\mu = 0.19$ or 0.38, S =1, 10 or 100 pounds, and N equal various values from 0 (the undamped case) to 50,000 pounds (with N appropriate to the other parameters selected).

The usual analysis is a series of computer runs with η_1, η_2, μ, and S constant and N varying from 0 to a selected upper limit in about ten steps. In each run solutions are obtained from a lower starting frequency to an upper ending frequency at fixed increments of the

frequency range. The range from 3,000 to 12,000 Hz has been evaluated in 100 Hz or 250 Hz increments in most runs because failures at the 8500 Hz mode when platform lockup occurs are of major interest. A few runs from 3,000 to 20,000 Hz have been processed and show that the friction damping is as effective at the second flexural mode of the free blade (18,000 Hz) as it is at the first flexural mode (4500 Hz).

Another controlling parameter of the friction damper performance is the blade to adjacent blade phasing. One would expect that when adjacent blades are in phase no damping by the interplatform friction dampers would occur. Conversely, maximum damping would be expected when the blades are out of phase. The interblade phase angle is controlled by δ_ν, the phase shift of the harmonic forcing function $S_\nu \cos(\omega t+\delta_\nu)$, and is input to each run as an array. The value of δ_ν, the phase angle of the νth blade for a phase tuned system is defined as:

$$\delta_\nu = \frac{2\pi E(\nu-1)}{n} \quad \text{and } \delta_2-\delta_1 = \delta_3-\delta_2, \text{ etc.} \tag{9}$$

where E is the engine order of the vibration mode and n is the total number of blades in the disk. For simplification n=64, the number of blades in a synthetic system has been used rather than n=63, the number of blades in the actual HPFTP first stage disk system. This allows for simplification and consequent cost savings in the computer runs for tuned systems. The blade to blade phase shift $(\delta_{\nu+1}-\delta_\nu)$, called phase angle θ, is defined as:

$$\theta = \frac{2\pi E}{n} \tag{10}$$

Thus $\theta = \pi = \frac{2\pi 32}{64} = \frac{2\pi}{2}$ a synthetic two-bladed tuned disk system. Similarly, $\theta = \frac{\pi}{2} = \frac{2\pi 16}{64} = \frac{2\pi}{4}$ a four-bladed system, $\theta = \frac{\pi}{4} = \frac{2\pi 8}{64} = \frac{2\pi}{8}$ an eight-bladed system, etc. In these tuned systems all blades experience identical deflections at the modal masses (D, A for m_1, m_2) and a minimally complex system is analyzed.

4.2.2 Analysis Results

A sample of program output data for the outboard modal mass deflection is presented in Figure 32 as a plot of amplitude D versus frequency and a similar set of data for the inboard modal mass deflection (amplitude A) is presented in Figure 33 for the same computer runs. These data sets arise from runs of the lumped mass program with the N variable successively assigned the values shown in the tables on the figures. The other system parameters for these runs were:

$$\theta = \pi/4 \text{ (8 bladed disk)}$$

$$S = 1.0 \text{ lb. } \cos(\omega t + \frac{2\pi(\nu-1)}{8})$$

$$\mu = 0.19$$

$$\eta_1 = 0.0005$$

$$\eta_2 = 0.005$$

and m_1, m_2, k_1, k_2 = HPFTP average blade modal values for the tuned first stage disk system. The η_2 value of 0.005 is an average value obtained during modal tests of blades hard-clamped in a broach block.

Figure 32 shows that amplitude D is reduced nearly three orders of magnitude when optimum damping occurs and that it is highly damped at 8500 Hz, even for large values of N, the damper normal force. Figure 33 shows that the amplitude A value at 8500 Hz is more than three orders of magnitude below the value for the free blade at 4500 Hz. This indicates that the Coulomb friction force supplies a substantial amount of damping for even very low amplitude platform motion. Figures 32 and 33 show also that the blade transitions very quickly from the 4500 Hz mode to the 8500 Hz mode at low values of N or $\mu N/S$ and that the minimum response of the blade occurs in this transition region near the midfrequency of the region.

Figure 34 shows the plots of the peak amplitude of the airfoil modal mass deflection relative to the platform-neck modal mass deflection (D-A) versus the ratio $\mu N/S$, the ratio of the friction force opposing platform motion to the blade forcing function amplitude.

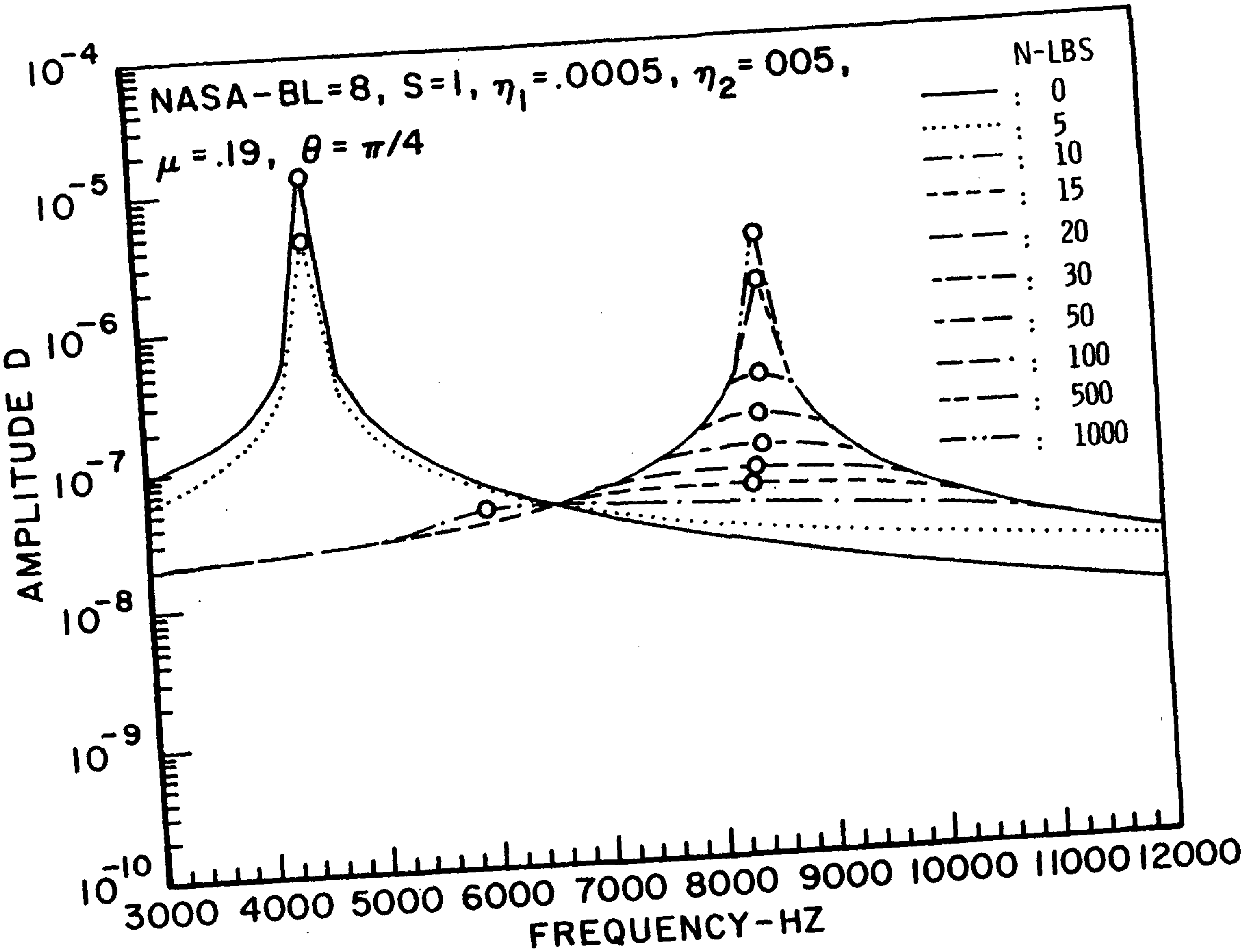


Figure 32 Amplitude of HPFTP Blade Modal Mass m_1 vs Frequency of Excitation

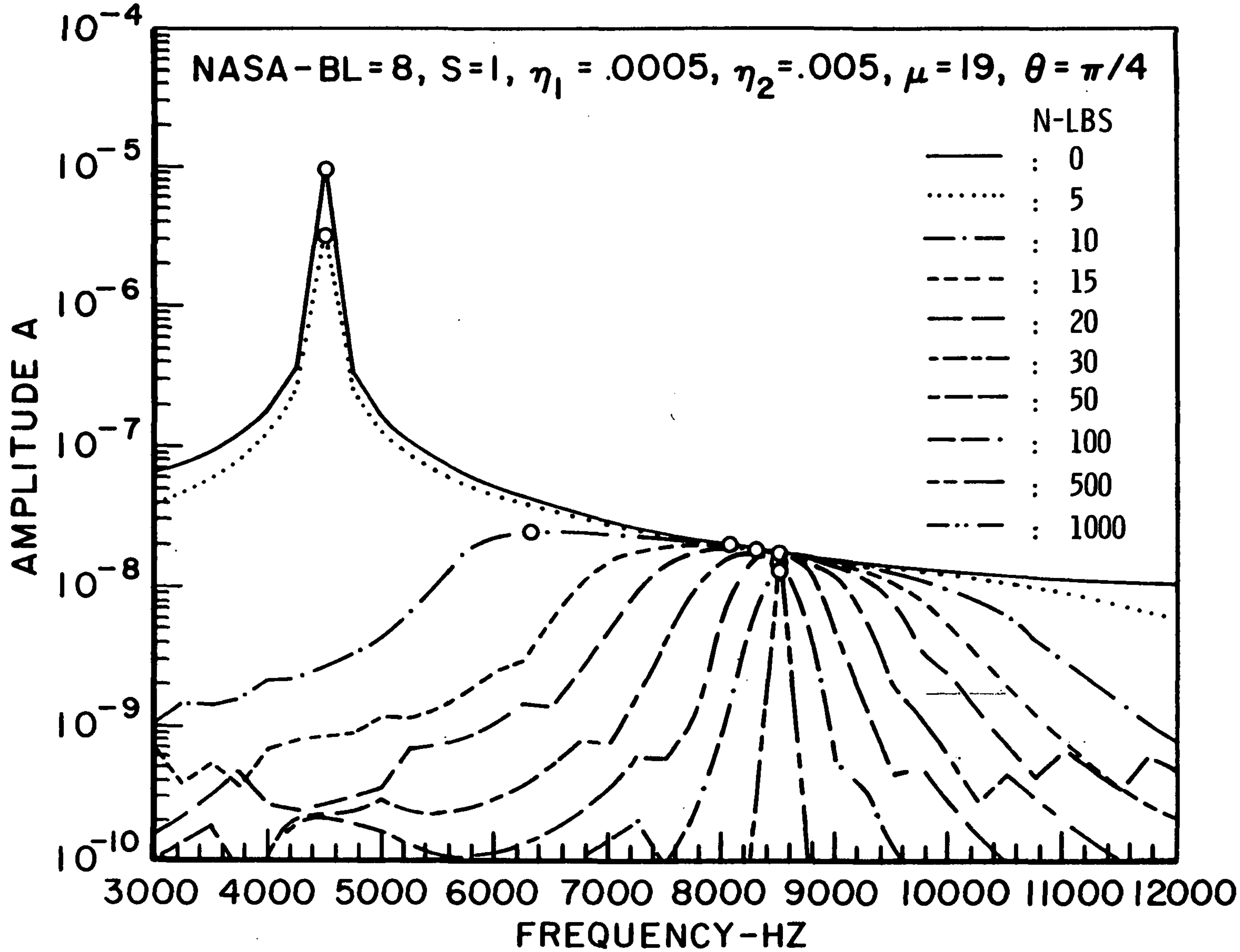


Figure 33 Amplitude of HPFTP Blade Modal Mass m_2 vs Frequency of Excitation

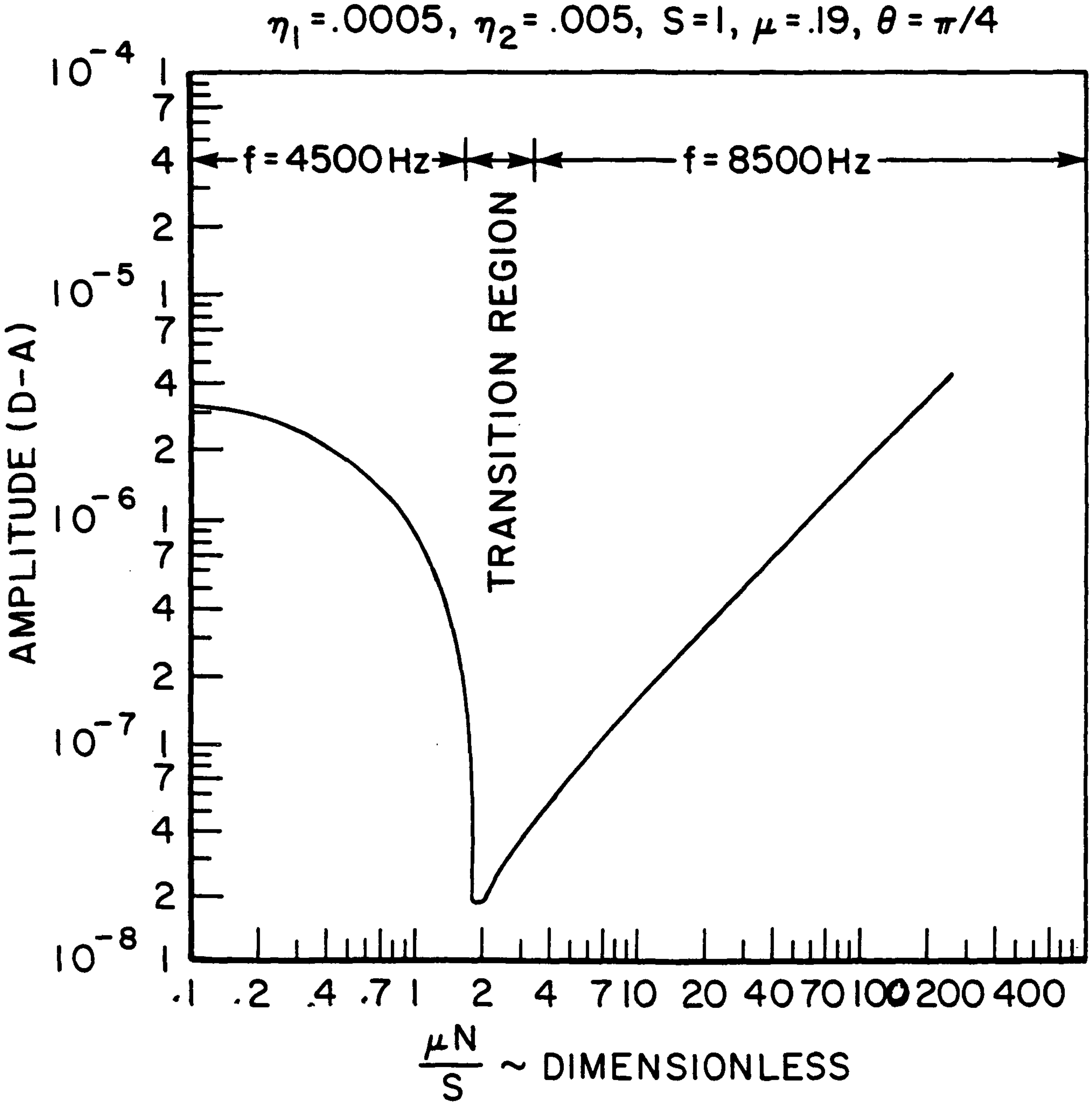


Figure 34 Amplitude of m_1 Relative to m_2 vs $\frac{\mu N}{S}$

This format of data is presented because the subject blade fails at the airfoil root with the platform. The data points selected for the Figure 34 plot are circled on Figures 32 and 33. Figures 32 and 33 show that the blade vibration amplitude is reduced optimally by friction damping at relatively low values of N. This fact is depicted very graphically in Figure 34. The transition region also is shown in Figure 34.

Figure 35 shows the effect of changes in hysteretic loss factors, η_1 and η_2 while all other parameters remain unchanged. Figure 35 shows that hysteretic damping is effective at the 4500 Hz modal frequency. After the friction damping forces the frequency into the transition region, however, the effect of the hysteretic damping becomes negligible.

Figure 36 shows the effect of variation of θ, the blade to blade phase angle. It is seen that a larger phase shift between blades up to an equivalent engine order (E) of vibration of n/2, produces a higher level of Coulomb friction damping, as expected. However, this is not usually a controllable parameter in an operational turbine, as may be true of most of the other parameters. The figure does show the characteristics of the curves for various engine orders of excitation, and provides useful design or evaluation information. It should be noted that the amplitude reduction possible is the same for all values of θ but at different values of $\mu N/S$.

Figure 37 shows the effect of variation of S, the forcing function amplitude. It should be noted that the three curves shown have identical shapes, but that for an order of magnitude increase in S the airfoil response amplitude (D-A) as a function of $\mu N/S$ increases by an order of magnitude. This shows the system to be linear with S and indicates that a unit curve (S=1) can be used to define a system. Then, the response amplitude can be scaled by the amplitude of S for any operating system having otherwise identical operating parameters.

All the previous data sets represent systems having a μ (the damper to platform coefficient of friction) of 0.19, an arbitrary

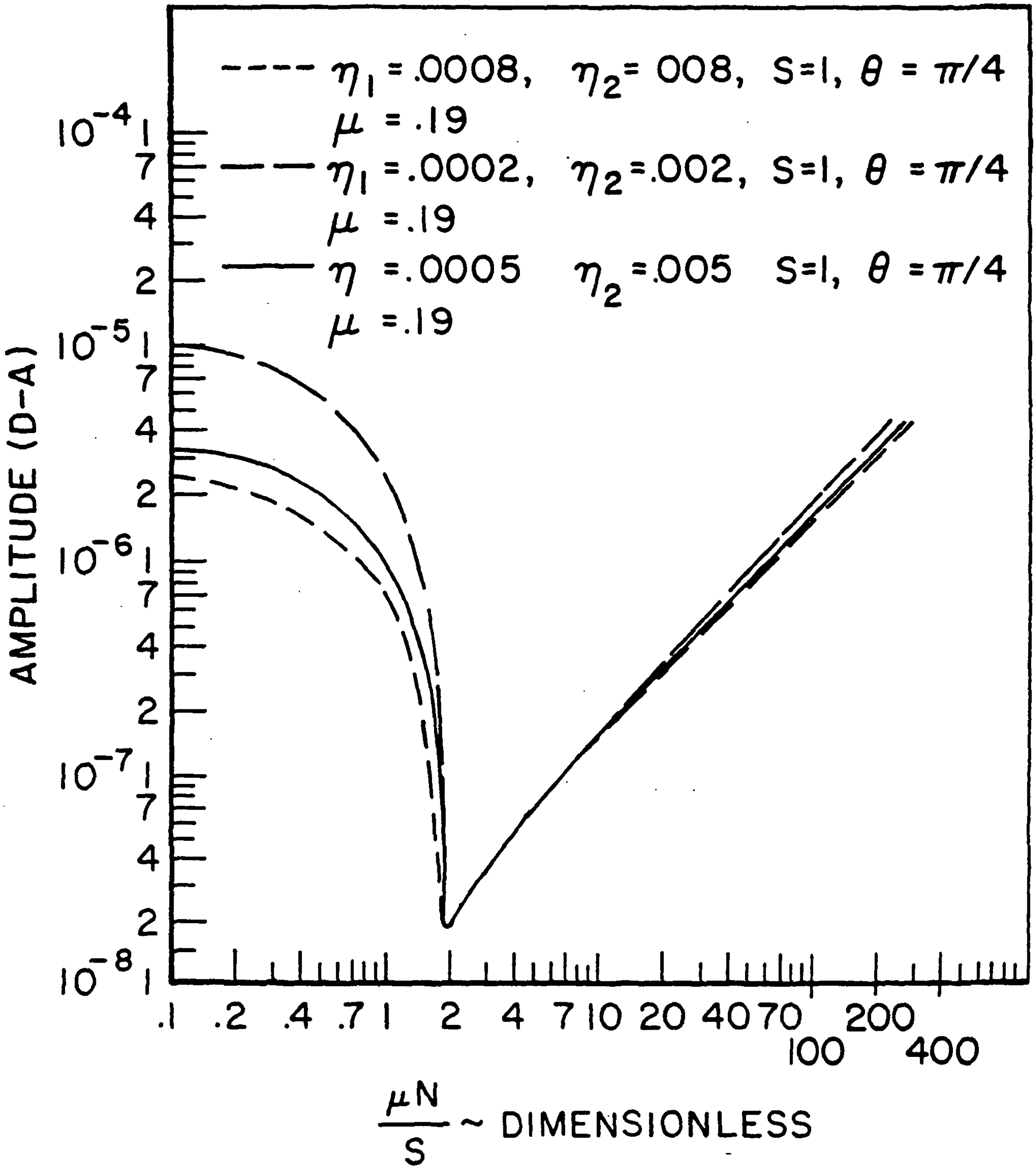


Figure 35 Amplitude of m_1 Relative to m_2 vs $\frac{\mu N}{S}$, Variation of η_1 and η_2

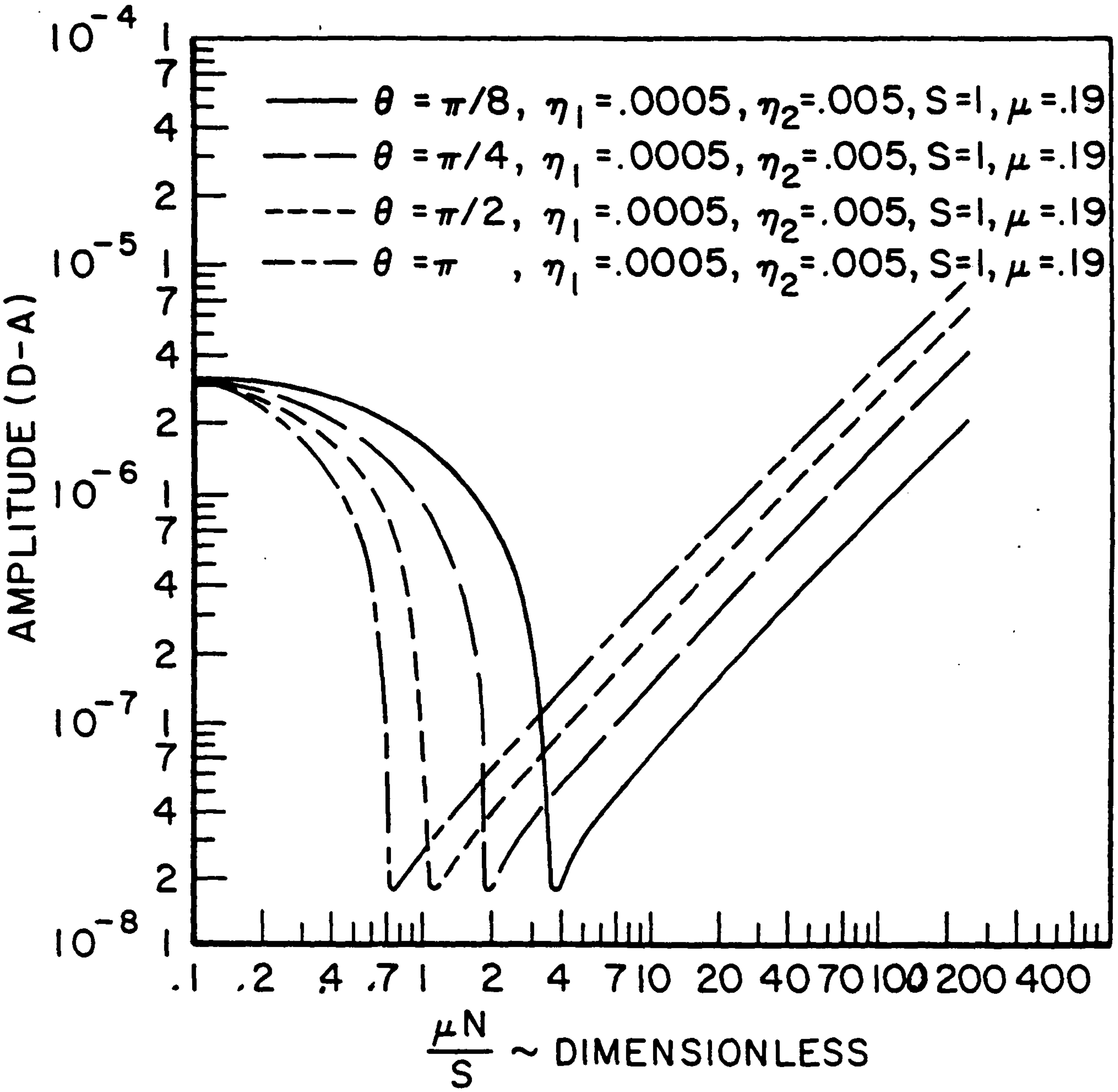


Figure 36 Amplitude of m_1 Relative to m_2 vs $\frac{\mu N}{S}$, Variation of θ

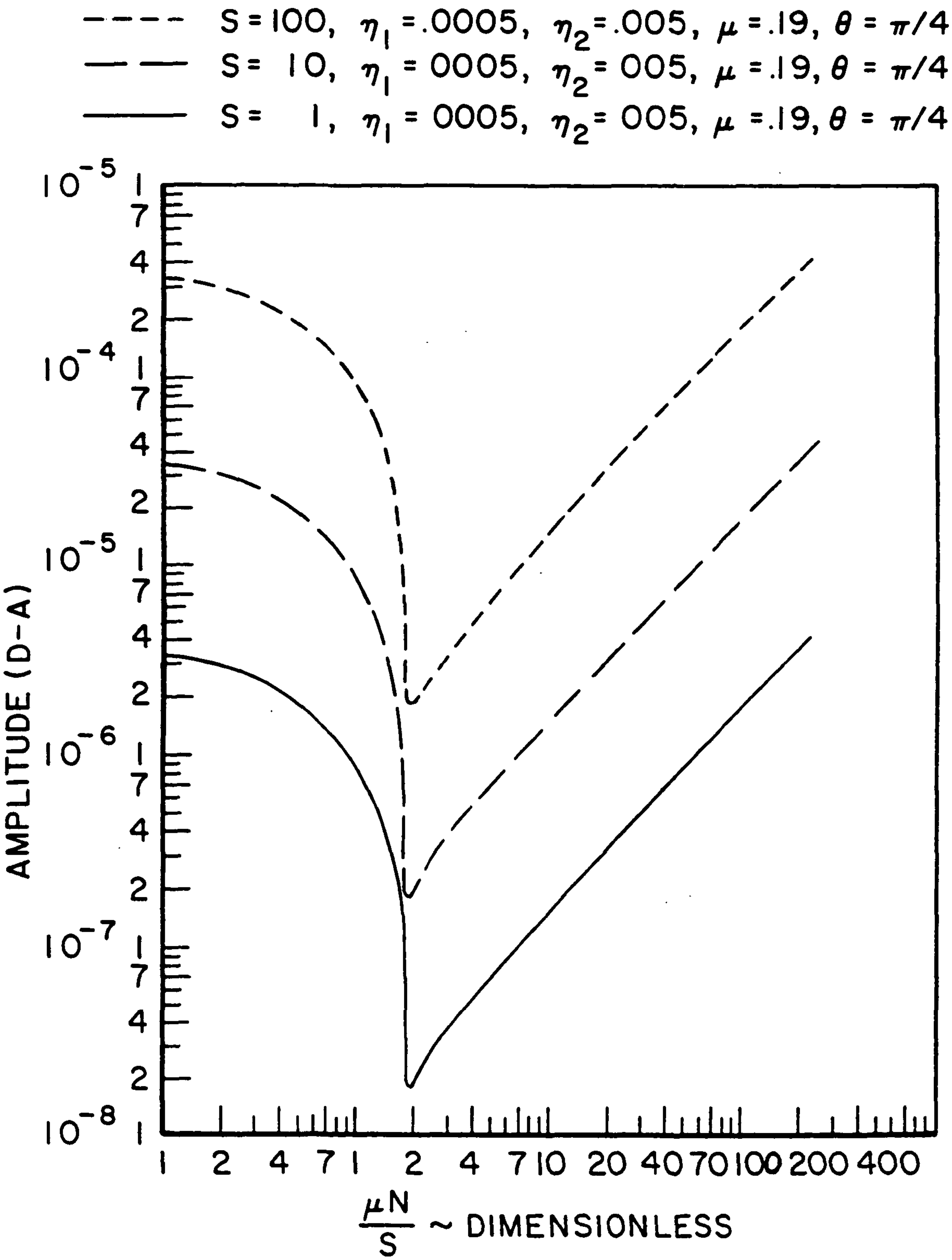


Figure 37 Amplitude of m_1 Relative to m_2 vs $\frac{\mu N}{S}$, Variation of S

value used previously by UDRI in the analysis for a previous program. A set of computer data was generated for a system having μ of 0.38, double the previous value. When a data point for $\mu N/S$ of $\frac{0.38 \times 5}{1}$ was plotted, the amplitude fell identically on that for $\frac{0.19 \times 10}{1}$, and similarly for all paired identical values of $\mu N/S$. Although μ and N are not independent variables, it is necessary to know their values and characteristics throughout the operating temperature-pressure-speed regime of a turbine. They may be amenable to some modification if design problems are encountered, or if operational fatigue problems are encountered later that necessitate modification of the system.

A weakness of this dynamic analysis is that the Coulomb friction model applies only over a limited lower part of the $\mu N/S$ range. The extent of Coulomb friction damping in the $\mu N/S$ range is affected by a number of friction surface parameters, the normal force, and by the vibration frequency and amplitude. Coulomb friction ends at some value of $\mu N/S$ and a regime of stick-slip friction begins. The stick-slip regime is followed by the end of slip, the lockup regime where friction ends and the surfaces are locked firmly together. Important questions for any friction damping system are:

1. Where do stick-slip begin and end? and
2. What are the effects of the stick-slip mode?

The consequence of platform lockup has been shown to be quite severe for the system under study, being almost immediate failure of the turbine blade. After the loss of friction damping the airfoil vibration amplitude at excited resonance modes is limited only by the blade material hysteretic loss factor (perhaps 0.0001) combined with whatever aerodynamic viscous damping is imparted to the blade by the driving fluid. The outboard blade section loss factor is estimated to be at least an order of magnitude less than that of the inboard blade section.

The consequences of stick-slip and lockup have been sketched on a copy of the Figure 34 graph as shown in Figure 38. The location of stick-slip onset, the breadth of the stick-slip range and the

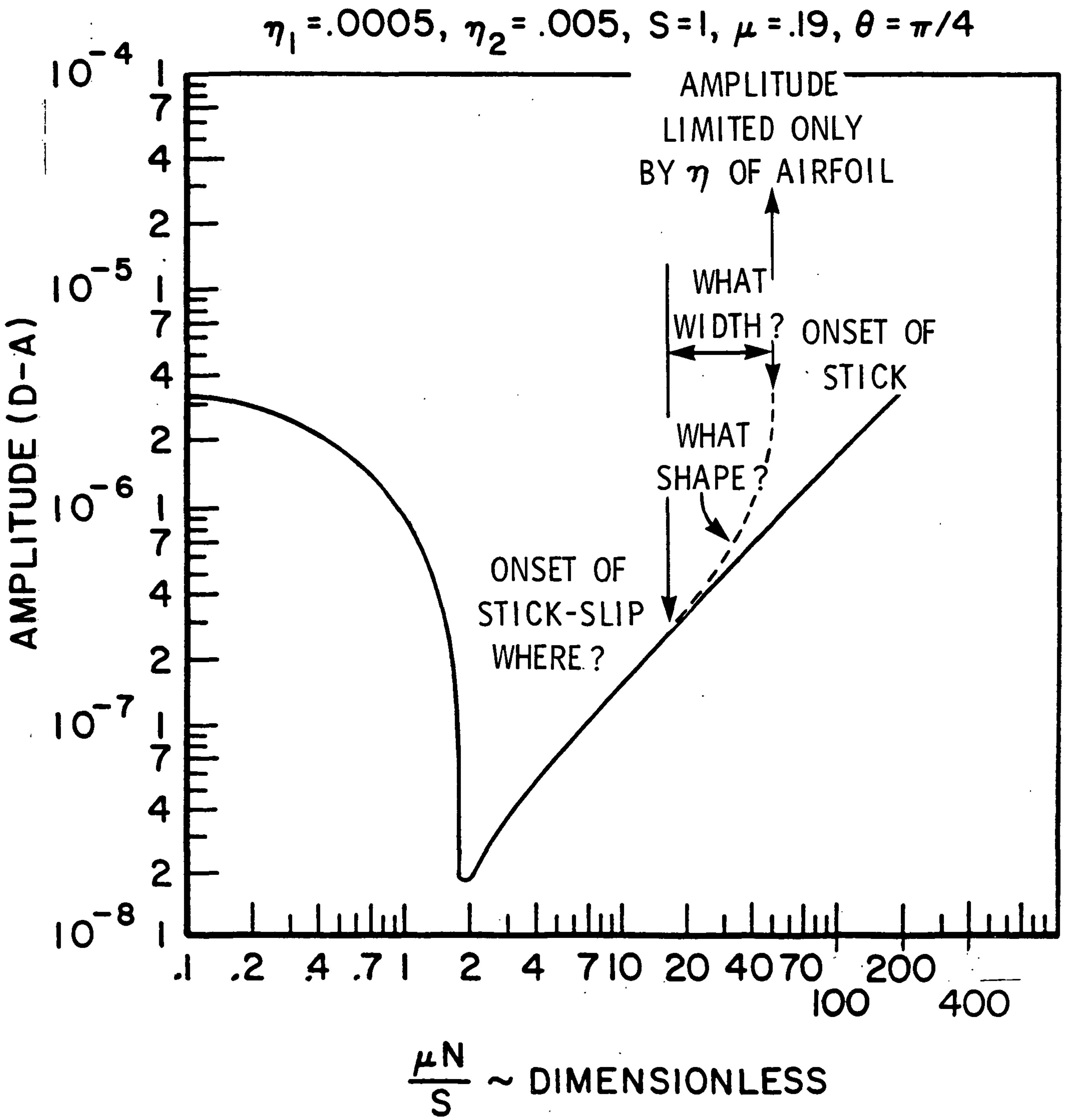


Figure 38 Amplitude of m_1 Relative to m_2 vs $\frac{\mu N}{S}$, Effects of Stick-slip and Stick

effective damping in that range may be critical to system survival. This information is likely to be indetermineable without a careful and costly test program. Such a program may yield only marginally satisfactory results. What is needed is instrumentation data during system operation. Acquisition of such data is still beyond the state of the art for some high performance systems, particularly small ones operating at high rotational speeds.

5 SUMMARY OF RESULTS

The results of the experimental and analytical program efforts are summarized below.

5.1 EXPERIMENTAL TEST RESULTS

The following results were obtained from the experimental test program.

1. The 0.56 gram production dampers did constrain the blade platforms at operational spin speeds and caused excessive airfoil-alone first flexural mode resonance vibration.
2. Blade root fixity and root damping varied over a considerable range as the disk was accelerated to operational spin speed. Root fixity, and consequently whole blade flexural resonance frequency, increased with increasing spin speed.
3. Lightweight experimental dampers eliminated both lower order flexural modes of the HPFTP blade for the conditions that occurred in the high speed spin test.
4. Both damped and undamped blades resonated in the first whole blade flexural resonance mode during the low speed spin test.
5. Magnetic excitation of the blades proved to be a feasible test technique for high speed spin condition tests.

5.2 ANALYTICAL STUDY RESULTS

The following results were obtained from the analytical study performed with the lumped mass computer program.

1. The parametric study data presented in Figures 35, 36 and 37 shows the effects of the variation of the coefficient of friction, normal force loading, excitation force amplitude, blade to blade phasing, and hysteretic damping on the HPFTP blade flexural vibration response up to 20 KHz if Coulomb friction damping is assumed for the platform friction dampers.
2. The ratio of the friction damper force amplitude (μN) to the excitation force amplitude (S) proved to have the most significant effect on the blade response.
3. The hysteretic damping of the blade does not affect its response appreciably in the $\mu N/S$ range where friction damping is effective.
4. Friction damping can reduce the response amplitude of the blade more than 99 percent at the minimum response point in the frequency transition region ($\mu N/S$ range of 0.5 to 10, dependent on θ).
5. Figure 37 shows that response curves for S=1 can be used to define this blade-damper system since the response amplitude scales linearly as a function of S (other values of S must be entered in the $\mu N/S$ parameter if linear scaling is used).
6. The data set shows the potential for airfoil root fatigue damage if platform fixity occurs because of high values of $\mu N/S$. Figure 38 shows that this potential will be realized if Coulomb friction becomes stick-slip friction or stick because of high values of the normal force (N).

6 CONCLUSIONS

We draw the following conclusions based on this turbine blade damping study.

1. The analytical results are in agreement with the high speed spin test results in that both show that the continuing high cycle fatigue problem in the HPFTP

turbine blade is due to restriction of blade platform motion by high friction forces of the platform friction dampers. This problem could be alleviated by reducing the normal force loading of the dampers or by reducing the damper to platform coefficient of friction.

2. The 14E excitation mode caused by pressure pulses off the front bearing support struts is the major cause of the damage at the airfoil-platform intersection because it occurs at the airfoil-alone first flexural resonance frequency for the 90 to 95 percent of engine operation time that the engine is at RPL.
3. The lumped mass analysis proved useful in defining the effects of the collective relationships of the five major parameters (coefficient of friction, damper to platform normal force, forcing function amplitude, blade to blade phasing, and hysteretic damping) that affect the vibration response of turbine blades which have blade platform friction dampers. It must be realized that physical factors affecting any one of the major parameters will change the vibration response of the blade.
4. The experimental spin tests showed that root fixity and root damping varied considerably with spin speed for the firtree root configuration of the blade that was tested. This caused wide variation in the amplitude and frequency of the whole-blade first flexural resonance mode of vibration.
5. Both the spin tests and analytical study showed that lower order blade flexural resonance modes of vibration can be controlled effectively by platform friction dampers if the dampers can be operated near their optimum effectivity conditions for the engine modes of operation that induce damaging vibrations of the blade.
6. The lumped mass computer analysis program is useful in defining design or evaluation and redesign parameters for blade platform friction dampers. The quantitative

results of the analytical program are dependent on the accuracy with which the input parameters to the program are known. Parametric studies similar to the one performed in this program for the HPFTP blade could be performed to define the range of flexural vibration responses of any blade-damper system.

7. Test studies to define the damper to blade coefficient of friction variation with normal load, temperature, vibration frequency and amplitude, and other local conditions (perhaps lubrication caused by fluid flows) would improve the accuracy of analytical studies a great deal. The definition of stick-slip operation and stick onset would be especially valuable if they could be obtained.
8. The definition of actual blade performance in an operating turbine is a critical need for improving blade system design and redesign accuracy. All test programs attempting to simulate turbine operations omit operational conditions that may be critical to the definition of the problem being studied.

7 RECOMMENDATIONS

The following recommendations are made with respect to the HPFTP first stage blade fatigue problem and to the generic area of blade-damper interactions.

1. We recommend further reduction of the weight of the present damper. We believe the best way to accomplish that weight reduction would be the substitution of beryllium for the present superalloy damper material. The present damper normal load (N) of 300 lb-f at RPL consists of 225 lb-f due to centrifugal force and 75 lb-f due to differential pressure loading. Using the lower density beryllium in the present damper configuration would reduce the centrifugal force loading by a factor of four (to about 55 lb-f) and the total normal force load would be more than halved to 130 lb-f.

2. We recommend continuing vigorous pursuit of instrumentation methodologies having the potential for providing accurate data defining the operation of turbine components throughout turbine operating regimes.
3. We recommend establishment of a test program to define the variations of the coefficient of friction for various material combinations used in blade-damper systems. The test program should include studies of the effects of all the physical parameters known to affect or suspected of affecting friction damper performance.
4. We recommend use of the lumped mass computer program as a cost-effective way of parametrically defining current or future blade platform friction damper systems' performance.
5. We recommend improving the lumped mass computer program to include variations in the coefficient of friction due to changes in other operational parameters if the definition of coefficient of friction variations becomes available.
6. We recommend further development of the finite element analysis (FEA) program for blade-damper system performance prediction if higher computer run time costs are tolerable. The FEA program analysis has the advantage that blade geometry is the major component of the input data set rather than the modal parameters that are rerequired (and which are not always determinable) for the lumped mass analysis. The FEA program also has the advantage that transient conditions can be resolved for systems where transient conditions are important.

REFERENCES

1. Sutton, R.F., "Nasa, High Speed Rotating Diagnostic Laboratory Testing, SSME High-pressure Fuel Turbopump Blade/Damper Evaluation," Rocketdyne Division of Rockwell International Report RSS-8626, November 2978.

2. Muszynska, A., D.I.G. Jones, T. Lagnese, and L. Whitford, "On Nonlinear Response of Multiple Blade Systems," Paper presented at 51st Shock and Vibration Symposium, Dan Diego, CA, October 1980, and published in Shock and Vibration Bulletin 51, Part 3, May 1981.

3. Scott, L.P., J.E. Pond, C.C. Myers, G.A. Teal, G.F. Lewis, and J.K. Robinson, "Assessment of HPFTP Turbine Blade Environment and Fatigue Life Study on the SSME, Volumes I and II," Lockheed Missiles and Space Company, Inc., Huntsville Research and Engineering Center Report LMSC-HREC TR D784198, May 1981.

4. Dickerson, E.O., "Turbine Blade Structural Dynamic Analysis," AIAA Paper 80-0782, 1980.

5. Dominic, R.J., Philip A. Graf, and B. Basava Raju, "Analytical and Experimental Investigation of Turbine Blade Damping - Final Report," University of Dayton Research Institute Report UDR-TR-82-39, August 1982. (AD-A120470, AFOSR-82-0911TR, or NTIS HC A04/MF A01).

6. Jones, D.I.G., "Vibrations of a Compressor Blade with Slip at the Root," AFWAL-TR-80-4003, April 1980.

APPENDIX A

UDR-TM-82-08

FORCED VIBRATIONS OF FRICTION-DAMPED BEAM STRUCTURES IN TWO DIMENSIONS

R. A. Brockman

March 1982

TABLE OF CONTENTS

SECTION

1. INTRODUCTION

This report describes a short development effort whose purpose is to define promising solution techniques for vibration problems involving friction damping in slender, flexible members. A primary application is the analytical study of root and platform damping in rotating engine blades.

The final product of the present effort is a computer program implementing several types of solutions which pertain to the evaluation of steady-state response in simple structures which can be represented adequately with beam-type finite elements. The next logical step is the detailed correlation of analytical and experimental data, to suggest those extensions and improvements which are necessary before the analytical predictions can be accepted with confidence.

2. ANALYSIS METHODOLOGY

Four types of analyses have been developed in the present study:

(1) static displacement solution,
(2) steady-state undamped harmonic solution,
(3) approximate steady-state friction-damping analysis, and
(4) transient vibration analysis with structural and friction damping.

Each of the four analysis types is based upon a particular special case of the dynamic equations of motion,

$$\underset{\sim}{M}\,\ddot{\underset{\sim}{X}} + \underset{\sim}{C}\,\dot{\underset{\sim}{X}} + \underset{\sim}{K}\,\underset{\sim}{X} = \underset{\sim}{F}(t) \tag{1}$$

The discrete system is obtained in all cases by means of the finite element method. The structure under consideration is represented by a collection of beam finite elements situated in the (X,Y) coordinate plane. These elements properly account for both axial and primary bending deformation, as well as ~~rotary~~ flexural rotary

inertia effects. Concentrated masses and axial rotational inertias may be added to complete the description of the system.

The basis of each analysis type, and any limitations, are summarized briefly in the following sections.

2.1 Static Displacement Solution

The static solution has been included in the computer analysis primarily as a facility for verifying the correctness of the data describing the finite element model. In the static analysis, inertial effects are neglected ($\dot{\underset{\sim}{X}} = \ddot{\underset{\sim}{X}} = \underset{\sim}{0}$) and the problem

$$\underset{\sim}{K}\underset{\sim}{X} = \underset{\sim}{F} \tag{2}$$

is solved. The triple-factor form of Gaussian elimination is used to solve (2), in the following steps:

Factorization:	$\underset{\sim}{K} = \underset{\sim}{L}\,\underset{\sim}{D}\,\underset{\sim}{L}^T$	(3)
Forward Solution:	$\underset{\sim}{L}\,\underset{\sim}{Z} = \underset{\sim}{F}$	(4)
Scaling:	$\underset{\sim}{D}\,\underset{\sim}{Y} = \underset{\sim}{Z}$	(5)
Backward Solution:	$\underset{\sim}{L}^T\underset{\sim}{X} = \underset{\sim}{Y}$	(6)

The same solution technique is used in the other three analysis branches whenever simultaneous equations must be solved.

2.2 Steady-State Harmonic Solution

The steady-state harmonic response serves as a baseline for the evaluation of the effects of damping (structural or friction) upon the behavior of the system. For this solution, all damping effects are neglected and a sinusoidal response is expected:

$$\underset{\sim}{X} = \underset{\sim}{U}\sin\omega t \tag{7}$$

Since the harmonic loading can be expressed as

$$\underset{\sim}{F}(t) = \underset{\sim}{F}_h \sin \omega t \tag{8}$$

the steady-state response amplitudes $\underset{\sim}{U}$ are determined by solving

$$(\underset{\sim}{K} - \omega^2 \underset{\sim}{M})\, \underset{\sim}{U} = \underset{\sim}{F}_h \tag{9}$$

The steady-state solution can be obtained whenever ω does not correspond exactly to a natural frequency of the model, where zeroes of $|\underset{\sim}{K} - \omega^2 \underset{\sim}{M}|$ occur.

2.3 Approximate Friction-Damping Solution

When friction forces are applied to the model, two types of friction-damping analysis are performed. The first of these is an approximate solution in which the behavior of the system is <u>assumed</u> to be sinusoidal at the forcing frequency:

$$\underset{\sim}{X}(t) = \underset{\sim}{U} \sin \omega t \tag{10}$$

Assumption (10) is reasonable whenever (a) the friction forces are small so that the motion is approximately sinusoidal, or (b) the friction forces are sufficiently large to cause "locking", effectively changing the nature of the support conditions. In the intermediate range of friction forces, "dead bands" in the system response assume a predominant role and assumption (10) is not applicable.

The approximate solution based upon (10) begins with an analysis similar to the steady-state harmonic solution, but having all of the friction joints <u>locked</u> at zero displacement. The internal forces required to maintain the locking constraints are computed, and the constraints are unlocked whenever

$$F_{int}^{(i)} > \mu_s N^{(i)} \tag{11}$$

If the amplitude of the tangential internal force does not satisfy (11), static friction is sufficient to lock the joint at all times and the joint remains locked.

Once the appropriate friction joints have been unlocked, another steady-state solution is performed (Equation 9) with the revised set of constraints. This second solution provides the displacement and velocity amplitudes needed to compute the work performed by friction forces.

Considering a quarter-cycle of motion, for which the initial conditions are

$$\underset{\sim}{X}(0) = \underset{\sim}{0} \quad ; \quad \dot{\underset{\sim}{X}}(0) = \omega \underset{\sim}{U} \quad ; \quad \ddot{\underset{\sim}{X}}(0) = \underset{\sim}{0} \tag{12}$$

the system gains potential energy $\frac{1}{2} \underset{\sim}{U}^T \underset{\sim}{K} \underset{\sim}{U}$ and loses kinetic energy $\frac{1}{2}\omega^2 \underset{\sim}{U}^T \underset{\sim}{M} \underset{\sim}{U}$, while the dissipation due to friction forces is

$$W_d = \int_0^{\pi/2\omega} \omega \underset{\sim}{U}^T \underset{\sim}{F}_f \, |\cos \omega t| \, dt = \frac{\pi}{2} \underset{\sim}{U}^T \underset{\sim}{F}_f \tag{13}$$

The work supplied by the harmonic forces is

$$W_h = \int_0^{\pi/2\omega} \underset{\sim}{U}^T \underset{\sim}{F}_h \omega \sin \omega t \cos \omega t \, dt = \frac{1}{2} \underset{\sim}{U}^T \underset{\sim}{F}_h \tag{14}$$

so that the balance of energy over this quarter-cycle gives

$$\frac{1}{2} \underset{\sim}{U}^T \left[(\underset{\sim}{K} - \omega^2 \underset{\sim}{M}) \underset{\sim}{U} + \pi \underset{\sim}{F}_f - \underset{\sim}{F}_h \right] = 0 \tag{15}$$

For arbitrary and independent u_i, the approximate response is then governed by

$$(\underset{\sim}{K} - \omega^2 \underset{\sim}{M}) \underset{\sim}{U} = \underset{\sim}{F}_h - \pi \underset{\sim}{F}_f \tag{16}$$

The steady-state velocity amplitudes $\omega \underset{\sim}{U}$ are used to determine the sense of $\underset{\sim}{F}_f$ at each friction joint. A final check is also made

to ensure that the character of the solution (in the form of relative signs of the nodal velocities) is not affected by $\underset{\sim}{F}_f$; such a change signals that the friction force is driving the response, and locking of the corresponding friction joint is indicated.

2.3 Transient Analysis

When sinusoidal forces and friction forces are both present, a complete transient analysis is performed to determine the damped response with greater precision. The transient solution is performed directly with Equation (1), with <u>all</u> applied forces (constant and harmonic) and <u>all</u> damping effects (friction, structural) included.

The time-dependent response is integrated using the Newmark method with $\beta = 1/4$ (trapezoidal rule), which is known to minimize numerical dissipation and period distortion. The appropriate finite difference formulas for the temporal discretization are:

$$\dot{\underset{\sim}{X}}_{t+\Delta t} = \dot{\underset{\sim}{X}}_t + \frac{\Delta t}{2}\left(\ddot{\underset{\sim}{X}}_t + \ddot{\underset{\sim}{X}}_{t+\Delta t}\right) \tag{17}$$

$$\underset{\sim}{X}_{t+\Delta t} = \underset{\sim}{X}_t + \Delta t\,\dot{\underset{\sim}{X}}_t + \frac{(\Delta t)^2}{4}\left(\ddot{\underset{\sim}{X}}_t + \ddot{\underset{\sim}{X}}_{t+\Delta t}\right) \tag{18}$$

Combining Equations (1), (17), and (18) yields the equation of motion in fully discrete form,

$$\left(\underset{\sim}{K} + \frac{2}{\Delta t}\underset{\sim}{C} + \frac{4}{\Delta t^2}\underset{\sim}{M}\right)\Delta\underset{\sim}{X} = \underset{\sim}{F}_{t+\Delta t} + 2\underset{\sim}{C}\dot{\underset{\sim}{X}}_t - \left(\underset{\sim}{M}\ddot{\underset{\sim}{X}} + \underset{\sim}{C}\dot{\underset{\sim}{X}} + \underset{\sim}{K}\underset{\sim}{X}\right)_t + \underset{\sim}{M}\left(2\ddot{\underset{\sim}{X}}_t + \frac{4}{\Delta t}\dot{\underset{\sim}{X}}_t\right) \tag{19}$$

where

$$\Delta\underset{\sim}{X} = \underset{\sim}{X}_{t+\Delta t} - \underset{\sim}{X}_t \tag{20}$$

The equation of motion is integrated directly in time, with the friction forces determined instantaneously by

$$F_f^{(i)} = \mu_d^{(i)} N^{(i)} \operatorname{sign}(\dot{X}_{rel}^{(i)}) \tag{21}$$

When structural damping is to be considered, the damping matrix $\underset{\sim}{C}$ is defined by

$$\underset{\sim}{C} = \frac{\eta}{\omega} \underset{\sim}{K} \tag{22}$$

where η is the material loss factor. This description of damping is appropriate for nearly-sinusoidal motions.

During the transient solution, a continuous monitoring is made of the most recent relative maxima and minima for each degree-of-freedom in the model. Experience has shown that the solution tends toward steady state after eight to ten cycles of motion, when the undamped steady motion is used to prescribe initial velocity conditions.

The present implementation of the transient dynamic solution does not perform locking and unlocking explicitly during the analysis, because of the potential for numerical instability in a non-iterative solution. Therefore, amplitudes computed with transient analysis branch may be overestimated, particularly in the vicinity of resonant frequencies.

3. COMPUTER PROGRAM INPUT/OUTPUT

The pilot computer program (UD-BLADE) developed to test the present analysis has been developed for the VAX 11/780 mini-computer. However, with minor modifications (described in the source code in comment lines) the code may be adapted to most scientific computers.

Execution of the program is controlled by the VAX/VMS control language procedure BLADE.COM; to initiate execution,

the command

@BLADE

is entered (or SUBMIT BLADE to execute as a background task). Input data, which are described in the following, should reside on the file BLADE.DAT prior to initiating execution. Two output files are generated by UD-BLADE:

BLADE.OUT : printer output

BLADE.HST : time history data for plotting

The output file BLADE.ØUT can be spooled to the line printer or listed at the terminal. It is quite short (two pages for a small analysis) and contains only an input summary, the static and/or steady-state displacement solutions, and a tabulation of the last relative minimum and maximum displacements attained during the transient solution. The BLADE.HST file is generated during a transient solution, and contains complete time histories for one or more user-selected "trace degrees of freedom"; typically these are degrees of freedom corresponding to locations where external or friction forces are applied. The time history data may be plotted on a Tektronix graphics terminal by entering the command

RUN BLADEPLOT

and answering a few simple questions.

Input data for UD-BLADE are described in the remainder of this report. The input is segmented into several blocks, each one preceded by a descriptive keyword for identification. Not all of the data blocks will be needed for a particular problem. Only those which are pertinent need be entered, and they may appear in any order on the input file.

Each item of input is described by a data type (I = integer, E = floating point, A = alphanumeric) in the descriptions to follow. It should be noted that all integer values must be right-justified in the input field provided. Floating point (real)

values may be entered either in F-format (e.g., 2.0876, 0.0428, etc.) placed anywhere in the data field, or in E-format (e.g., 1.2E4, 0.2E-2, etc.) with the exponent right-justified. Data fields which are left blank will be interpreted as zero (0 or 0.0 as appropriate).

A force/length/time system is used throughout the program, and any consistent set of units may be used to define a problem. Note that mass is therefore a derived quantity, with units of (force x time^2/length). For example, if the lbf-in-sec system of units is used, the proper mass unit is lbf-sec^2/in, and a mass per unit length would be expressed in lbf-sec^2/in^2.

TITLE Data Block

(Optional)

Line	Columns	Data	Type	Description
1	1-80	TITLE	A	Alphanumeric Problem Title

PARAMETERS Data Block

(Required for a steady-state or transient solution)

Columns	Data	Type	Description
1-10	FREQB	E	Beginning frequency (cps)
11-20	FREQI	E	Frequency increment
21-30	FREQE	E	Final frequency
31-40	DAMP	E	Damping (loss) factor
41-45	INTRANS	I	Flag for Transient solution = 0, no trnasient solution (s) = 1, do transient solution at each Frequency
46-50	NTRACE	I	Number of Trace Degrees of Freedom for Plotting File Output (Max = 5)
51-55	ITRACE (1)	I	First Trace Deg. of Freedom
.	.	.	. . .
.	.	.	. . .
.	.	.	. . .
.	.	.	. . .
71-75	ITRACE (5)	I	Fifth Trace Deg. of Freedom

NOTES:

1. If FREQI and FREQE are omitted (blank), a single solution will be performed at W = FREQB
2. ITRANS = O is suggested for multiple-frequency runs
3. ITRACE (1) will be written as output to the frequency response plot file BLADE.FRQ for each type of solution performed, at each forcing frequency value.

COORDINATES Data Block

(Required)

Line	Columns	Data	Type	Description
(Typ.)	1-5	NODE	I	Node Point Number
	6	(blank)		
	7	IBX	I	Constraint code for X-displacement
	8	IBY	I	Constraint code for Y-displacement
	9	IBR	I	Constraint code for rotation
	10	(blank)		
	11-20	X(NODE)	E	X-Coordinate
	21-30	Y(NODE)	E	Y-Coordinate

----Terminate this block with a blank line ----

Notes:

1. Repeat the input above for each node to be defined.
2. Constraint codes (IBX, IBY, IBR) are defined as 0 if the degree-of-freedom is free, 1 if constrained.

ELEMENTS Data Block

(Required)

Line	Columns	Data	Type	Description
(Typ.)	1-5	IEL	I	Element Number
	6-10	NODE1	I	First Connected Node
	11-15	NODE2	I	Second Connected Node
	16-25	E	E	Elastic Modulus
	26-35	A	E	Beam Cross-Sectional Area
	36-45	BI	E	Bending Moment of Inertia
	46-55	RHO	E	Density (Mass per unit length)

---- Terminate this block with a blank line ----

Notes:

1. Repeat the input above for each beam element to be defined.
2. If E, A, BI, or RHO is omitted for an element, the value given for the first element input will be used.
3. Mass densities should be entered in a force-length-time system of units. If w represents the weight density of the material (weight/unit volume), then RHO=wA/g, where A is the beam area and g is the gravitational acceleration.

MASS Data Block

(Optional)

Line	Columns	Data	Type	Description
(Typ.)	1-5	NODE	I	Node Number for Concentrated Mass
	6-15	CMASS	E	Concentrated Mass
	16-25	CINERT	E	Rotational Inertia

---- Terminate this block with a blank line ----

Notes:

1. Repeat the input above for each lumped mass/inertia to be defined.
2. Masses and rotational inertias must be entered in a force-length-time system of units.

FORCES Data Block

(Required)

Line	Columns	Data	Type	Description
(Typ.)	1-5	NODE	I	Node Number at which Force is Applied
	6-10	IDIR	I	Force Direction (1 = X, 2 = Y, 3 = Moment)
	11-15	ITYPE	I	Force Type =0, Static Load =1, Harmonic Load
	16-25	FORCE	E	Force Value (or Amplitude)

---- Terminate this block with a blank line ----

Notes:

1. Static forces (ITYPE=0) are included in both the static solution and transient solution. Harmonic forces are included in all solutions except the static (steady-state, steady-state with friction, and transient).
2. The value of FREQ in the PARAMETERS data block defines the frequency of <u>all</u> harmonic (ITYPE=1) forces.

CONTACT Data Block

(Optional)

Line	Columns	Data	Type	Description
(Typ.)	1-5	NODE	I	Node Number at which Contact Occurs
	6-10	IDIR	I	Direction of Normal Force (1=X, 2=Y)
	11-20	FN	E	Normal Force Magnitude (>0)
	21-30	SCF	E	Static Friction Coefficient
	31-40	DCF	E	Dynamic Friction Coefficient

---- Terminate this block with a blank line ----

Notes:

1. Repeat the input above for each point at which normal and friction forces are applied.
2. The static coefficient of friction must be larger than the dynamic coefficient of friction.

File Output from BLADE Program

1. BLADE.OUT (Printed Output)

   Contains a summary of problem input, and one page (or so) of amplitude output for each forcing frequency

2. BLADE.HST (Plotting File)

   Contains time histories for <u>all</u> "trace degrees of freedom" (up to 5) specified, computed in the transient solution branch. For multiple-frequency runs, this file is written separately at each frequency value.

3. BLADE.FRQ (Plotting File)

   Contains amplitude-versus-frequency data from

   (a) steady-state solution

   (b) steady-state solution with friction

   (c) transient solution (if performed)

- Both plotting files are written in a format which can be used with Tom Held's PLOT program.

SAMPLE INPUT AND OUTPUT

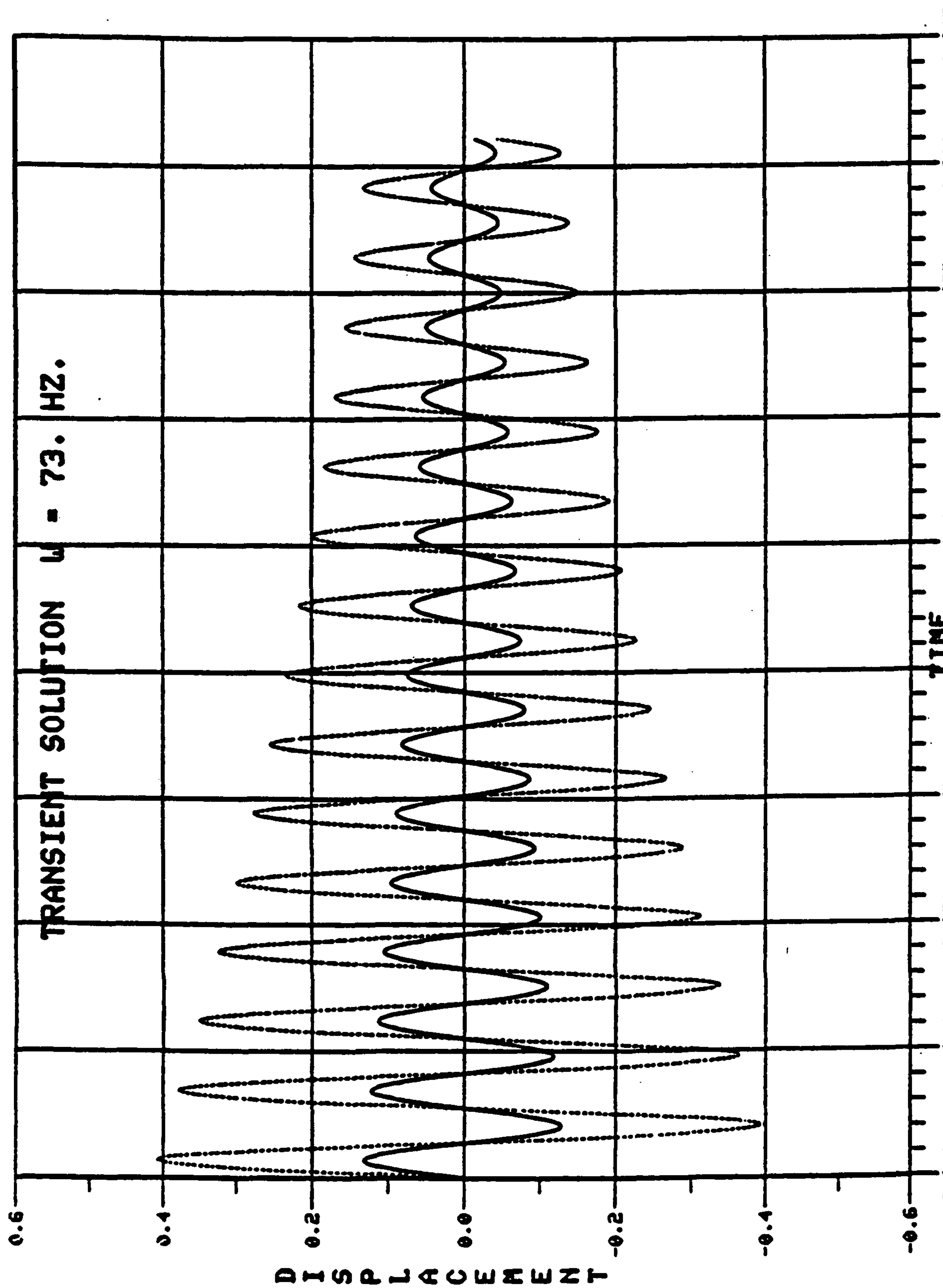
```
$ TYPE BLADE.DAT
TITL
TEST CASE FOR 2-D BEAM WITH FRICTION - CANTILEVER WITH TIP LOAD
COOR
    1 111        0.        0.
    2            5.        0.
    3           10.        0.

ELEM
    1    1    2      1.E7        1.       0.01  0.000420
    2    2    3      1.E7        1.       0.01  0.000420

FORC
    3    2    1         1.

PARA
     73.00     0.020    2    5    8
CONT
    2    1         4.        .31        .30

END
$
```

```
********************************************
 S T E A D Y - S T A T E   M O T I O N
   ( W I T H   F R I C T I O N )
********************************************
```

NODE	X--DISPLACEMENT	Y--DISPLACEMENT	(X,Y)--ROTATION
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.000000E+00	0.000000E+00	-0.784728E-04
3	0.000000E+00	0.889809E-03	0.230372E-03

```
*************************************************************
LAST REL. MAX / MIN DISPLACEMENTS IN TRANSIENT SOLUTION
*************************************************************
```

NODE	--X-DISPL(MIN,MAX)--		--Y-DISPL(MIN,MAX)--		--ROTATION(MIN,MAX)--	
1	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
2	0.000E+00	0.000E+00	-0.412E-01	0.429E-01	-0.151E-01	0.145E-01
3	0.000E+00	0.000E+00	-0.128E+00	0.133E+00	-0.189E-01	0.197E-01


TRANSIENT SOLUTION W = 73. HZ.
TIME
DISPLACEMENT
0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
0.225
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6

APPENDIX B

UDR-TR-84-38

USERS MANUAL FOR A COMPUTER PROGRAM FOR DYNAMIC RESPONSE ANALYSIS OF BLADED SYSTEMS

M. L. Soni
T. W. Held

University of Dayton Research Institute
300 College Park Avenue
Dayton, OH 45469

May 1984

ABSTRACT

This report presents a Users Manual for a computer program for lumped parameter modeling of the dynamic response of solid friction coupled bladed-disk systems.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

1. INTRODUCTION

In Reference 1, Muszynska, et al. derived a two parameter lumped mass model for frequency response analysis of multiple blade systems with blade-to-blade and blade-to-disk coupling. This report presents a Users Manual to the computer program implementing the above model. In the model derived in Reference 1, it is assumed that the blade sections between the root and the platform and the section above the platform have the same hysteretic damping factors. In Reference 2 it has been argued that separate loss factors should be used for the two sections of the blade. The governing equations and the computer program presented in this report permit the specification of different damping factors for the blade sections.

2. MATHEMATICAL MODEL

Figure 1 shows the lumped parameter model of the multiple blade system with blade-to-blade and blade-to-disk coupling. It consists of n numbers of two lumped mass-spring systems; n being the number of blades. Interblade coupling is provided through a spring and a solid friction damper, where blade-to-disk coupling is through a solid friction damper only. The blades are treated clamped to rigid disk. All springs are hysteretically (complex-stiffness) damped.

The governing equations of the system are

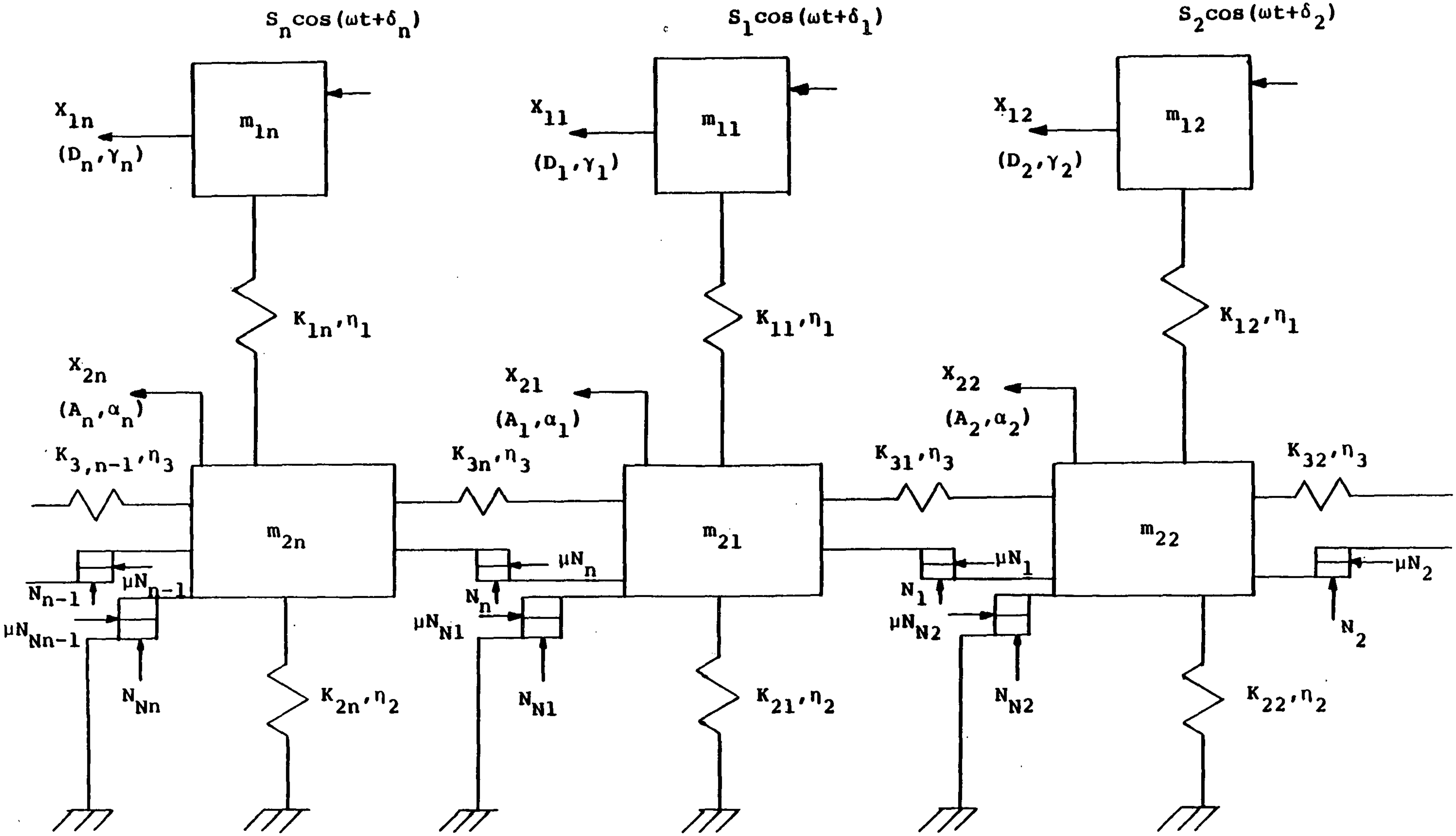


Figure 1. Lumped Parameter Model of Bladed Disk System.

$$
\begin{aligned}
&m_{1\nu}\ddot{X}_{1\nu} + K_{1\nu}(X_{1\nu}-X_{2\nu}) + \frac{K_{1\nu}\eta_1}{\omega}(\dot{X}_{1\nu}-\dot{X}_{2\nu}) = S_\nu \cos(\omega t+\delta_\nu) \\
&m_{2\nu}\ddot{X}_{2\nu} - K_{1\nu}X_{1\nu} + \frac{K_{1\nu}\eta_1}{\omega}(\dot{X}_{2\nu}-\dot{X}_{1\nu}) \\
&\quad + \frac{K_{2\nu}\eta_2}{\omega}\dot{X}_{2\nu} + \frac{K_{3\nu}\eta_3}{\omega}(\dot{X}_{2\nu}-\dot{X}_{2,\nu+1}) \\
&\quad + \frac{K_{3,\eta-1}\eta_3}{\omega}(\dot{X}_{2\nu}-\dot{X}_{2,\nu-1}) - K_{3\nu}X_{2,\nu+1} \\
&\quad - K_{3,\nu-1}X_{2,\nu-1} \\
&\quad + \mu N_\nu R_2 \operatorname{sgn}(\dot{X}_{2\nu}-\dot{X}_{2,\nu+1}) + \mu N_{N\nu} R_1 \operatorname{sgn}(\dot{X}_{2\nu}) \\
&\quad + \mu N_{\nu-1} R_2 \operatorname{sgn}(\dot{X}_{2\nu}-\dot{X}_{2,\nu-1}) + (K_{1\nu}+K_{2\nu} \\
&\qquad + K_{3\nu} + K_{3,\nu-1})\, X_{2\nu} = 0 \qquad\qquad \nu=1,2,\ldots,n
\end{aligned}
\tag{1}
$$

where $m_{1\nu}$, $m_{2\nu}$, and $K_{1\nu}$, $K_{2\nu}$ are the discrete masses and springs representing the blade ν, $K_{3\nu}$ is the blade-to-blade coupling stiffness, η_1, η_2, and η_3 are the hysteretic loss factors of the three springs, μ is the coefficient of solid friction, N_ν and $N_{N\nu}$ are normal forces related to the rotational speed of the bladed disk, R_1 and R_2 are associated connecting coefficients, S_ν and ω are respectively the amplitude and frequency of the harmonic excitation force, $X_{1\nu}$, $X_{2\nu}$ are the circumferential deflections of the two masses, and S_ν represents the time lag of a traveling wave excitation around the disk system (with $\delta_\nu=2\pi(\nu-1)/n$). The above equations are essentially the same as those derived in Reference 1 except for the different loss factors associated with springs $K_{1\nu}$ and $K_{2\nu}$.

Following Reference 1, the response of the system to time harmonic excitation is also assumed time harmonic in the form

$$X_1 = D_\nu \cos(\omega t+\gamma_\nu)$$

$$X_2 = A_\nu \cos(\omega t+\alpha_\nu) \tag{2}$$

where D_ν, A_ν and γ_ν, α_ν are, respectively, the amplitudes and phase angles of the response displacements of the two masses.

With the substitution of Equation 3 in the Equations 1 and 2 and the approximation

$$\operatorname{sgn}\left(C_1 \sin(\omega t+\beta_1) - C_2 \sin(\omega t+\beta_2)\right)$$

$$\cong \frac{4}{\pi} \frac{C_1 \sin(\omega t+\beta_1)-C_2 \sin(\omega t+\beta_2)}{\sqrt{C_1{}^2+C_2{}^2-2C_1C_2 \cos(\beta_1-\beta_2)}} \tag{3}$$

leads to a nonlinear algebraic matrix equation of the form

$$\underline{\underline{P}}\,(\underline{Z}) = \underline{Q} \tag{4}$$

where $\underline{\underline{P}}$ is a symmetric nonpositive matrix of order 2nx2n and $\underline{Z}$ and $\underline{Q}$ are vectors of length 2n, and are given as follows:

$$\underset{\sim}{Z} = \begin{bmatrix} z_1 \\ z_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ z_{m-1} \\ z \end{bmatrix} = \begin{bmatrix} A_1 \operatorname{Cos}\alpha_1 \\ -A_1 \operatorname{Sin}\alpha_1 \\ A_2 \operatorname{Cos}\alpha_2 \\ -A_2 \operatorname{Sin}\alpha_2 \\ \cdot \\ \cdot \\ A_n \operatorname{Cos}\alpha_n \\ -A_n \operatorname{Sin}\alpha_n \end{bmatrix} \tag{5}$$

$$
\underset{\sim}{Q} = \begin{bmatrix} Q_1 \\ Q_2 \\ \cdot \\ \cdot \\ \cdot \\ Q_{m-1} \\ Q_m \end{bmatrix} = \begin{bmatrix} S_1(R_{11}\text{Cos}\delta_1+R_{21}\text{Sin}\delta_1) \\ S_2(R_{11}\text{Sin}\delta_1+R_{21}\text{Cos}\delta_1) \\ \cdot \qquad \cdot \\ \cdot \qquad \cdot \\ \cdot \qquad \cdot \\ S_n(R_{1n}\text{Cos}\delta_n+R_{2n}\text{Sin}\delta_n) \\ S_n(R_{1\ }\ \text{Sin}\delta_n+R_{2n}\text{Cos}\delta_n) \end{bmatrix} \tag{6}
$$

$$
\underset{\sim}{P} = \left[\begin{array}{cc|cc|cc|cc}
P_{11} & P_{12}+W_1+W_n+V_1 & -K_{31} & -\eta_3K_{31}-W_1 & 0 & & -K_{3n} & -\eta_3K_{3n}-W_n \\
P_{12}+W_1+W_n+V_1 & -P_{11} & -\eta_3K_{31}-W_1 & K_{31} & & & -\eta_1K_{3n}-W_n & K_{3n} \\
\hline
-K_{31} & -\eta_3K_{31}-W_1 & P_{21} & P_{22}+W_2+W_1+V_2 & -K_{32} & -\eta_3K_{32}-W_2 & & \\
-\eta_3K_{31}-W & K_{31} & P_{22}+W_2+W_1+V_2 & -P_{21} & -\eta_3K_{32}-W_2 & K_{32} & 0 & \\
\hline
 & & -K_{32} & -\eta_3K_{32}-W_2 & P_{31} & P_{32}+W_3+W_2+V_3 & & \\
 & & & \ddots & & & & \\
\hline
 & 0 & -K_{3,n-2} & -\eta_3K_{3,n-2}-W_{n-2} & P_{n-1,1} & P_{n-1,2}+W_{n-1}+W_{n-2}+V_{n-1} & -K_{3,n-1} & -\eta_3K_{3,n-1}-W_{n-1} \\
 & & -\eta_3K_{3,n-2}-W_{n-1} & K_{3,n-2} & P_{n-1,2}+W_{n-1}+W_{n-2}+V_{n-1} & -P_{n-1,1} & -\eta_3K_{3,n-1}-W & K_{3,n-1} \\
\hline
-K_{3n} & -\eta_3K_{3n}-W_n & & & -K_{3,n-1} & -\eta_3K_{3,n-1}-W_{n-1} & P_{n1} & P_{n2}+W_n+W_{n-1}+V_n \\
-\eta_3K_{3n}-W_n & K_{3n} & 0 & & -\eta_3K_{3,n-1}-W_{n-1} & K_{3,n-1} & P_{n2}+W_n+W_{n-1}+V_n & -P_{n1}
\end{array}\right] \tag{7}
$$

where

$$
P_{\nu 1} = K_{2\nu} + K_{3\nu} + K_{3,\nu-1} - m_{2\nu}\omega^2 - \frac{K_{1\nu}m_{1\nu}\omega^2[(1+\eta_1^2)K_{1\nu}-m_{1\nu}\omega^2]}{(K_{1\nu}-m_{1\nu}\omega^2)^2 + (K_{1\nu}\eta_1)^2}
$$

$$
P_{\nu 2} = \eta_2K_{2\nu} + \eta_3(K_{3\nu} + K_{3,\nu-1}) + \frac{\eta_1K_{1\nu}\ m_{1\nu}^2\omega^4}{(K_{1\nu}-m_{1\nu}\omega^2)^2 + (K_{1\nu}\eta_1)^2}
$$

The normal load coefficients are

$$R_{1\nu} = \frac{\left(1 + \eta_1^2 - \frac{m_{1\nu}}{K_{1\nu}} \omega^2\right)}{\left[\left(1 - \frac{m_{1\nu}}{K_{1\nu}} \omega^2\right)^2 + \eta_1^2\right]} \tag{8}$$

$$R_{2\nu} = \frac{\left(\frac{m_{1\nu}}{K_{1\nu-1}} \eta_1 \omega^2\right)}{\left[\left(1 - \frac{m_{1\nu}}{K_{1\nu}} \omega^2\right)^2 + \eta_1^2\right]} \tag{9}$$

$$W_\nu = \frac{4\mu N_\nu R_{2\nu}}{\pi G_\nu} \tag{10}$$

and

$$V_\nu = \frac{4\mu N_{N\nu} R_{1\nu}}{\pi A_\nu} \tag{11}$$

where

$$A_\nu = \sqrt{Z_{2\nu-1}^2 + Z_{2\nu}^2} \tag{12}$$

$$G_\nu = \sqrt{(Z_{2\nu} - Z_{2\nu+2})^2 + (Z_{2\nu-1} - Z_{2\nu+1})^2} \tag{13}$$

with $\nu = 1, 2, \ldots n$; and $n+1 = 1$, $n+2 = 2$.

3. SOLUTION PROCEDURE

The equations (4), as noted earlier, are nonlinear in the solution vector $\underline{Z}$, are are solved iteratively [1].

A sequence $Z^{(\sigma)}$ of approximate solutions is obtained which satisfies

$$\underset{\sim}{P}^{(\sigma-1)} \underset{\sim}{Z}^{(\sigma)} = \underset{\sim}{Q}; \quad \sigma = 1, 2, \ldots$$

where

$$\underset{\sim}{P}^{(\sigma-1)} = \underset{\sim}{P}(\underset{\sim}{Z}^{(\sigma-1)}); \quad \sigma = 2, 3, \ldots$$

Initially, $\underset{\sim}{P}^{(o)}$ is calculated with $W_\nu = W_\nu^{(o)} = 0$, and $V_\nu = V_\nu^{(o)}$ for $\nu = 1,\ldots,m$ and the system

$$\underset{\sim}{P}^{(o)}\underset{\sim}{Z} = \underset{\sim}{Q}$$

is solved for $\underset{\sim}{Z} = \underset{\sim}{Z}^{(1)}$, $A_\nu^{(1)}$, $G_\nu^{(1)}$, $W_\nu^{(1)}$, $V_\nu^{(1)}$, and finally $\underset{\sim}{P}^{(1)}$ are then computed and

$$\underset{\sim}{P}^{(1)}\underset{\sim}{Z} = \underset{\sim}{Q} \tag{14}$$

solved for $\underset{\sim}{Z}^{(2)}$. The process continues with each iteration obtained by a standard linear equation solving subroutine. The particular algorithm utilizes UL factorization with iterative refinement. The process terminates when any one of the following conditions is satisfied:

$$\begin{aligned} &(1) \quad \max_\nu \left| Z_\nu^{(\sigma)} - Z_\nu^{(\sigma-1)} \right| < \varepsilon_1 \\ &(2) \quad \max_\nu \left| \frac{Z_\nu^{(\sigma)} - Z_\nu^{(\sigma-1)}}{Z_\nu^{(\sigma-1)}} \right| < \varepsilon_2 \\ &(3) \quad \sigma = \text{ITMAX (maximum number of iterations)} \end{aligned} \tag{15}$$

(ε_1, ε_2 - specified values defining the precision of results).

The amplitudes and phases of the solution (3) are finally obtained as

$$A_\nu = \sqrt{(Z_{2\nu-1})^2 + (Z_{2\nu})^2} \tag{16}$$

$$D_\nu = \{S_\nu^2 + A_\nu^2 K_{1\nu}^{\ 2}\ (1+\eta_1^2) + 2A_\nu S_\nu K_{1\nu}[\mathrm{Cos}(\delta_\nu - \alpha_\nu)$$

$$+ \eta_1 \mathrm{Sin}(\delta_\nu - \alpha_\nu)]\}^{1/2}[(K_{1\nu} - m_{1\nu}\omega^2)^2 \tag{17}$$

$$+ (K_{1\nu}\eta_1)^2]^{-1/2}$$

$$\alpha_\nu = \text{arc tan}\left(\frac{-Z_{2\nu}}{Z_{2\nu-1}}\right) \tag{18}$$

$$\gamma_\nu = \text{arc tan}\ [\{S_\nu(K_{1\nu} - m_{1\nu}\omega^2)\ \mathrm{Sin}\ \delta_\nu - K_{1\nu}\eta_1$$

$$\cdot \mathrm{Cos}\ \delta_\nu] + A_\nu K_{1\nu}\mathrm{Sin}\ \alpha_\nu[K_{1\nu}(1 + \eta_1^2) - m_{1\nu}\omega^2]$$

$$- A_\nu \mathrm{Cos}\ \alpha_\nu m_{1\nu}K_{1\nu}\eta_1\omega^2\}/\{S_\nu(K_{1\nu} - m_{1\nu}\omega^2)\ \mathrm{Cos}\ \delta_\nu \tag{19}$$

$$+ K_{1\nu}\eta_1\ \mathrm{Sin}\ \delta_\nu] + A_\nu K_{1\nu}\ \mathrm{Cos}\ \alpha_\nu[K_{1\nu}(1 + \eta_1^2) - m_{1\nu}\omega^2]$$

$$+ A_\nu\ \mathrm{Sin}\ \alpha_\nu m_{1\nu}K_{1\nu}\eta_1\omega^2\}]$$

$$\nu = 1, \ldots, n$$

4. COMPUTER PROGRAM

The analysis procedure of the preceding section is implemented in a computer program BLADE. The program is written in FORTRAN 77 and is operational on the VAX/VMS minicomputer. The program requires IMSL [3] equation solving subroutine LEQ2S.

The program consists of three parts: Interactive input data generator program DATAFORM, the response analysis program BLADE, and the interactive plotting program BPLOT.

Input to the program consists of specifications of the spring-mass and damping parameters of the blades and amplitude, phase and frequency of blade tip excitation forces. The program output consists of the displacement and acceleration amplitudes and phase

angles of the two masses. This information is printed out and also written to separate files for subsequent plotting for each blade and excitation frequencies. The following files are created.

File Name	Contents
AXXOUT.DAT	Displacement Amplitude of Mass 2
DXXOUT.DAT	Displacement Amplitude of Mass 1
ALPOUT.DAT	Phase Angle of Mass 2
GAMOUT.DAT	Phase Angle of Mass 1
AACCEL.DAT	Acceleration Amplitude of Mass 2
DACCEL.DAT	Acceleration Amplitude of Mass 1

The following describes the command procedure to use the data generator, analysis, and plotting programs.

I. Create Executable Programs:

a. Response Analysis Program

```
$ FOR BLADE
$ LINK BLADE,[IMSL]IMSLDB.OLB/LIBRARY
```

b. Data Formatter

```
$ FOR DATAFORM
$ LINK DATAFORM
```

c. Plot Program

```
$ FOR BPLOT
$ LINK BPLOT,[TEKTRN]PLOT1Ø.OLB/LIBRARY
```

where [IMSL]IMSLDB.OLB and the [TEKTRN]PLOT1Ø.OLB are the local, installation dependent names of the IMSL and plotting program libraries.

II. Execute Command Procedure PROC.COM to

a. Specify data file (see Appendix A for the format) and to use or interactively create (optional) a new data file.

b. Create a command procedure file JCL.COM.

III. Execute BPLOT to obtain plotted output. BPLOT is an interactive program and prompts a user to input necessary data. Some notes on running the program are given in Appendix B.

A listing of the programs and the command procedure is given in Appendix C.

5. EXAMPLE PROBLEM

The program operation is demonstrated in an example problem described below. A tuned bladed disc system is chosen with

No. of blades	:	n		$= 4$
Mass 1	:	$m_{1\nu}$	$\nu = 1,2,3,4$	$= 0.02$ lbm
Mass 2	:	$m_{2\nu}$	$\nu = 1,2,3,4$	$= 0.07975$ lbm
Spring Stiffness	:	$K_{1\nu}$	$\nu = 1,2,3,4$	$= 5705000.$ lbf/in
Spring Stiffness	:	$K_{2\nu}$	$\nu = 1,2,3,4$	$= 2859000.$ lbf/in
Spring Stiffness	:	$K_{3\nu}$	$\nu = 1,2,3,4$	$= 0.0$
Excitation Force	:	S_{ν}	$\nu = 1,2,3,4$	$= 1.0$ lbf
Phase Angle	:	δ_{ν}	$\nu = 1,2,3,4$	$= 0,\pi/2,\pi,3\pi/2$
Loss Factors	:	η_1		$= .002$
Loss Factors	:	η_2		$= .02$
Loss Factors	:	η_3		$= 0.$
Coefficient of Friction	:	μ		$= 0.19$
Interfacial Normal Force	:	$N_{N\nu}$		$= 0.$
Interfacial Normal Force	:	N_{ν}		$= 100.$ lbf

Frequency response is calculated for discrete frequencies ranging from 3000 Hz to 10000 Hz with a resolution of 250 Hz.

The following pages present (a) the terminal session for creating input data and JCL files, (b) the partial listing of the output of the response analysis, and (c) the terminal session for plotting response quantities. The plotted output is shown in Figure 2.

It should be noted that the plot program allows plotting of data generated in several separate response analysis runs thus for example, the above problem may be run for several different values of the parameter N_{ν}. In each run a set of plot files AXXOUT.DAT, DXXOUT.DAT, etc. are created. By specifying the number of such files created, in the plotting program, the relevant

response quantity for different values of N_ν may be plotted simultaneously. Figure 3 shows one such plot where 11 different values of N_ν are used and response amplitude versus frequency curve for each value of N_ν is plotted.

TERMINAL SESSION FOR CREATING
INPUT AND JCL FILES

```
$ PROC.COM

****************************************************************
*                                                              *
*               BLADE BATCH INPUT PROCEDURE                    *
*               ---------------------------                    *
*          TERMINAL SESSION :  22-MAY-1984 17:55
*          -------------------------------------
*                                                              *
****************************************************************

  CREATE NEW DATA FILE? (Y/N).......: Y

  ENTER NEW DATA FILE NAME.TYPE.....: BLADE.DAT;1

          DATA FILE FORMATTER
          -------------------

 TO INTERRUPT OR MODIFY,
 ENTER -999 FOR ANY INPUT VALUE.

 ENTER TITLE (50 CHAR MAX).......: NASA-BL-BL:4

 ENTER NUMBER OF BLADES..........: 4

 ENTER OMEGAO, OMGEND, DOMEGA....: 3000,10000,250

 ENTER ITMIN, ITMAX, EPS, EPS1,
 ZLIM............................: 2,30,1.E-09,5.E-02,1.E-12

 ENTER PROGRAM RUN-TIME LIMIT....: 2500

 ENTER ETA1, ETA2, ETA3, MU.......: .002,.02,0.,.19

 ENTER GRAVITATIONAL CONSTANT....: 384.

 FOR M1     : ARE ALL VALUES EQUAL ? (Y/N)..: Y

 ENTER M1    ( 1).........: .02

 FOR M2     : ARE ALL VALUES EQUAL ? (Y/N)..: Y

 ENTER M2    ( 1).........: .007975

 FOR K1     : ARE ALL VALUES EQUAL ? (Y/N)..: Y

 ENTER K1    ( 1).........: 5.705E+07

 FOR K2     : ARE ALL VALUES EQUAL ? (Y/N)..: Y
```

```
ENTER K2   ( 1).........: 2.859E+07

FOR K3    : ARE ALL VALUES EQUAL ? (Y/N)..: Y

ENTER K3   ( 1).........: 0.0

FOR S     : ARE ALL VALUES EQUAL ? (Y/N)..: Y

ENTER S    ( 1).........: 1.0

FOR DELTA : ARE ALL VALUES EQUAL ? (Y/N)..: N

ENTER DELTA( 1).........: 0.0

ENTER DELTA( 2).........: 1.57080

ENTER DELTA( 3).........: 3.14159

ENTER DELTA( 4).........: 4.71239

FOR NN    : ARE ALL VALUES EQUAL ? (Y/N)..: 0.0

FOR NN    : ARE ALL VALUES EQUAL ? (Y/N)..: Y

ENTER NN   ( 1).........: 0.0

FOR N     : ARE ALL VALUES EQUAL ? (Y/N)..: Y

ENTER N    ( 1).........: 100.

ANY MODIFICATIONS ?  (Y/N) .....: N

WRITING TO FILE...

DATA FORMAT PROGRAM IS DONE.


FORTRAN STOP

   WRITING COMMAND FILE...

   COMMAND FILE WRITTEN TO : JCL.COM;3

  DISPLAY PROCEDURE HERE? (Y/N).....: Y

$ SET VERIFY
$!
$!   BLADE BATCH INPUT PROCEDURE
$!      USER : [BLADE]
$!   SESSION : 22-MAY-1984 17:55
$!
$ ON ERROR THEN GOTO TERMINUS
$ SET DEF [BLADE]
$   ASSIGN/USER_MODE BLADE.DAT;1 FOR009
$   ASSIGN          JCL.LOG SYS$PRINT
$!
$   RUN BLADE
$!
$!
$ TERMINUS:
$  SET NOON
$  DELETE FOR002.DAT;0
$  SET ON
```

```
$  SET NOVERIFY
$  EXIT

   TERMINAL SESSION ENDED.

$
```

DATA FILE CREATED :

```
TYPE BLADE.DAT;1

NASA-BL-BL:4
        4
  3000.000       10000.00       250.0000
        2             20  9.9999997E-10  5.0000001E-02  1.0000000E-12
  2500.000
 2.0000001E-03  2.0000000E-02  0.0000000E+00  0.1900000
  384.0000
M1      ALL
 2.0000000E-02
M2      ALL
 7.9750000E-03
K1      ALL
 5.7050000E+07
K2      ALL
 2.8590000E+07
K3      ALL
 0.0000000E+00
S       ALL
  1.000000
DELTA
 0.0000000E+00   1.570800       3.141590       4.712390
NN      ALL
 0.0000000E+00
N       ALL
  100.0000
$
```

PROGRAM EXECUTION:

The program is executed by issuing a command

$ SUBMIT JCL.COM

which creates an output file JCL.LOC described below

PROGRAM OUTPUT:

A partial listing of the program output is given in the following pages.

INPUT PARAMETERS

```
PROBLEM DESCRIPTION : NASA-BL-BL:4

NUMBER OF BLADES........:    4

INPUT FREQUENCIES :

   1.  INITIAL OMEGA......: 3.00000E+03

   2.  FINAL OMEGA........: 1.00000E+04

   3.  DELTA OMEGA........: 2.00000E+02

ITERATION PARAMETERS :

   1.  MINIMUM ITERATIONS :    2

   2.  MAXIMUM ITERATIONS :   30

   3.  EPS................: 1.00000E-09

   4.  EPS1...............: 5.00000E-02

   5.  ZLIM...............: 1.00000E-12

PROGRAM RUN-TIME LIMIT  :     2500.0

MODEL PARAMETERS :

   1.  ETA1...............: 2.00000E-03

   2.  ETA2...............: 2.00000E-02

   3.  ETA3...............: 0.00000E+00

   4.  MU.................: 1.90000E-01

GRAVITATIONAL CONSTANT  : 3.84000E+02
```

INPUT PARAMETERS (CONT'D)

```
M1    = 2.00000E-02 2.00000E-02 2.00000E-02 2.00000E-02

M2    = 7.97500E-03 7.97500E-03 7.97500E-03 7.97500E-03

K1    = 5.70500E+07 5.70500E+07 5.70500E+07 5.70500E+07

K2    = 2.85900E+07 2.85900E+07 2.85900E+07 2.85900E+07

K3    = 0.00000E+00 0.00000E+00 0.00000E+00 0.00000E+00

S     = 1.00000E+00 1.00000E+00 1.00000E+00 1.00000E+00

DELTA= 0.00000E+00 1.57080E+00 3.14159E+00 4.71239E+00

NN    = 0.00000E+00 0.00000E+00 0.00000E+00 0.00000E+00

N     = 1.00000E+02 1.00000E+02 1.00000E+02 1.00000E+02
```

 BLADE DYNAMICS PROGRAM - NASA-BL-BL:4 ***** PAGE 3 *****

STEP= 1 OMEGA= 3000.00000 ITERATIONS= 4 ABS. Z DIFF= 6.96266158E-11 REL. Z DIFF= 9.66690062E-01

BLADE	A	ALPHA	D	GAMMA	Z(2*I-1)//Z(2*I)	Z1(2*I-1)//Z1(2*I)
1	2.41084319E-12	-90.000	2.00224247E-08	-0.139	0.000000000000E+00	0.000000000000E+00
	2.23069393E-06		1.85262573E-02		2.410843193548E-12	7.203735023471E-11
2	2.41047259E-12	0.000	2.00224247E-08	89.861	2.410472590721E-12	7.203699495625E-11
	2.23035102E-06		1.85262573E-02		0.000000000000E+00	0.000000000000E+00
3	2.39917464E-12	90.000	2.00224247E-08	179.861	0.000000000000E+00	0.000000000000E+00
	2.21989730E-06		1.85262573E-02		-2.399174637420E-12	-7.202579040600E-11
4	2.39954524E-12	180.000	2.00224247E-08	-90.139	-2.399545238962E-12	-7.202614590590E-11
	2.22024021E-06		1.85262573E-02		0.000000000000E+00	0.000000000000E+00

STEP= 2 OMEGA= 3200.00000 ITERATIONS= 4 ABS. Z DIFF= 8.16540317E-11 REL. Z DIFF= 9.66019648E-01

BLADE	A	ALPHA	D	GAMMA	Z(2*I-1)//Z(2*()	Z1(2*I-1)//Z1(2*I)
1	2.88544225E-12	-90.000	2.04227775E-08	-0.143	0.000000000000E+00	0.000000000000E+00
	3.03767185E-06		2.15002384E-02		2.885442252760E-12	8.453934817827E-11
2	2.88497887E-12	0.000	2.04227775E-08	89.857	2.884978870454E-12	8.453890308275E-11
	3.03718402E-06		2.15002384E-02		0.000000000000E+00	0.000000000000E+00
3	2.87223217E-12	90.000	2.04227775E-08	179.857	0.000000000000E+00	0.000000000000E+00
	3.02376483E-06		2.15002384E-02		-2.872232173581E-12	-8.452626385036E-11
4	2.87269555E-12	180.000	2.04227775E-08	-90.143	-2.872695554234E-12	-8.452670921046E-11
	3.02425266E-06		2.15002384E-02		0.000000000000E+00	0.000000000000E+00

STEP= 3 OMEGA= 3400.00000 ITERATIONS= 4 ABS. Z DIFF= 9.84913899E-11 REL. Z DIFF= 9.65276132E-01

BLADE	A	ALPHA	D	GAMMA	Z(2*I-1)//Z(2*()	Z1(2*I-1)//Z1(2*I)
1	3.55837981E-12	-90.000	2.08669355E-08	-0.148	0.000000000000E+00	0.000000000000E+00
	4.22900937E-06		2.47996197E-02		3.558379808503E-12	1.020496201407E-10
2	3.55777186E-12	0.000	2.08669355E-08	89.852	3.557771859504E-12	1.020490347757E-10
	4.22828685E-06		2.47996197E-02		0.000000000000E+00	0.000000000000E+00
3	3.54302975E-12	90.000	2.08669355E-08	179.852	0.000000000000E+00	0.000000000000E+00
	4.21076636E-06		2.47996196E-02		-3.543029745180E-12	-1.020344196216E-10
4	3.54363769E-12	180.000	2.08669355E-08	-90.148	-3.543637691966E-12	-1.020350053127E-10
	4.21148889E-06		2.47996196E-02		0.000000000000E+00	0.000000000000E+00

SAMPLE PLOTTING SESSION:

```
RUN BPLOT

 MAX INPUT FILES................: 20
 HOW MANY INPUT FILES?..........: !

 MAX INPUT FILES................: 20
 HOW MANY INPUT FILES?..........: 1

 ENTER FILENAME NO.  1 .........: AXXOUT.DAT

 ENTER CHAR PER SECOND..........: 960

 ENTER TEK TERMINAL TYPE........: 4014

 DISPLAY OUTPUT SUMMARY ? (Y/N).: Y

 OUTPUT SUMMARY FOR FILE  1 :
 ----------------------------
 FILE NAME : AXXOUT.DAT

    RUN TITLE : NASA-BL-BL:4

    NUMBER OF Y COLUMNS.........:     4

    X VALUE RANGE :
    X MIN.................:   3.0000E+03
    X MAX.................:   1.0050E+04

    POINTS RANGED FROM       1 TO     142

    MINIMUM Y VALUES ON RANGE :
    Y MIN.................:   1.9028E-12
    IN COLUMN.............:            3

    MAXIMUM Y VALUES ON RANGE :
    Y MAX.................:   1.6335E-08
    IN COLUMN.............:            1

    TOTAL POINTS IN RANGE..:         142

 SEMI-LOGARITHMIC GRID OK? (Y/N): Y

 FOR FILE  1 :
 MIN X FOR RUN :   3.00000E+03
 MAX X FOR RUN :   1.00500E+04

 ENTER X-MIN, X-MAX FOR PLOT....: 3000,10000

 DISPLAY OUTPUT SUMMARY ? (Y/N).: Y

 OUTPUT SUMMARY FOR FILE  1 :
 ----------------------------
 FILE NAME : AXXOUT.DAT

    RUN TITLE : NASA-BL-BL:4

    NUMBER OF Y COLUMNS.........:     4
```

```
   X VALUE RANGE:
   X MIN..................:   3.0000E+03
   X MAX..................:   1.0000E+04

   POINTS RANGED FROM       1 TO     141

   MINIMUM Y VALUES ON RANGE :
   Y MIN..................:   1.9028E-12
   IN COLUMN..............:            3

   MAXIMUM Y VALUES ON RANGE :
   Y MAX..................:   1.6335E-08
   IN COLUMN..............:            1

   TOTAL POINTS IN RANGE..:          141

ENTER Y-MIN, Y-MAX FOR PLOT....: 1.9E-12,1.7E-08

ENTER NUMBER OF CURVES
FOR FILE  1 ...................: 1

FOR THIS FILE,
ENTER COLUMN NO. FOR CURVE  1 .: 1

DRAW A LEGEND? (Y/N)...........: Y

CURVE ( 1) DESCRIPTION.....: S1/N100

WHEN PLOT IS DONE, SET CURSOR
FOR UPPER LEFT CORNER OF LEGEND
BOX AND TYPE A SINGLE CHARACTER.


ENTER X-AXIS LABEL (MAX 30 CH) : EXCITATION FREQUENCY (HZ)

ENTER Y-AXIS LABEL (MAX 30 CH) : AMPLITUDE A INCHES

ENTER PLOT TITLE...............: AMPLITUDE FREQUENCY RESPONSE
```

PLOTTED OUTPUT:

Figure 2 shows the plot generated using the above procedure.

Figure 3 shows the plotted output for multiple values of normal load N_ν.

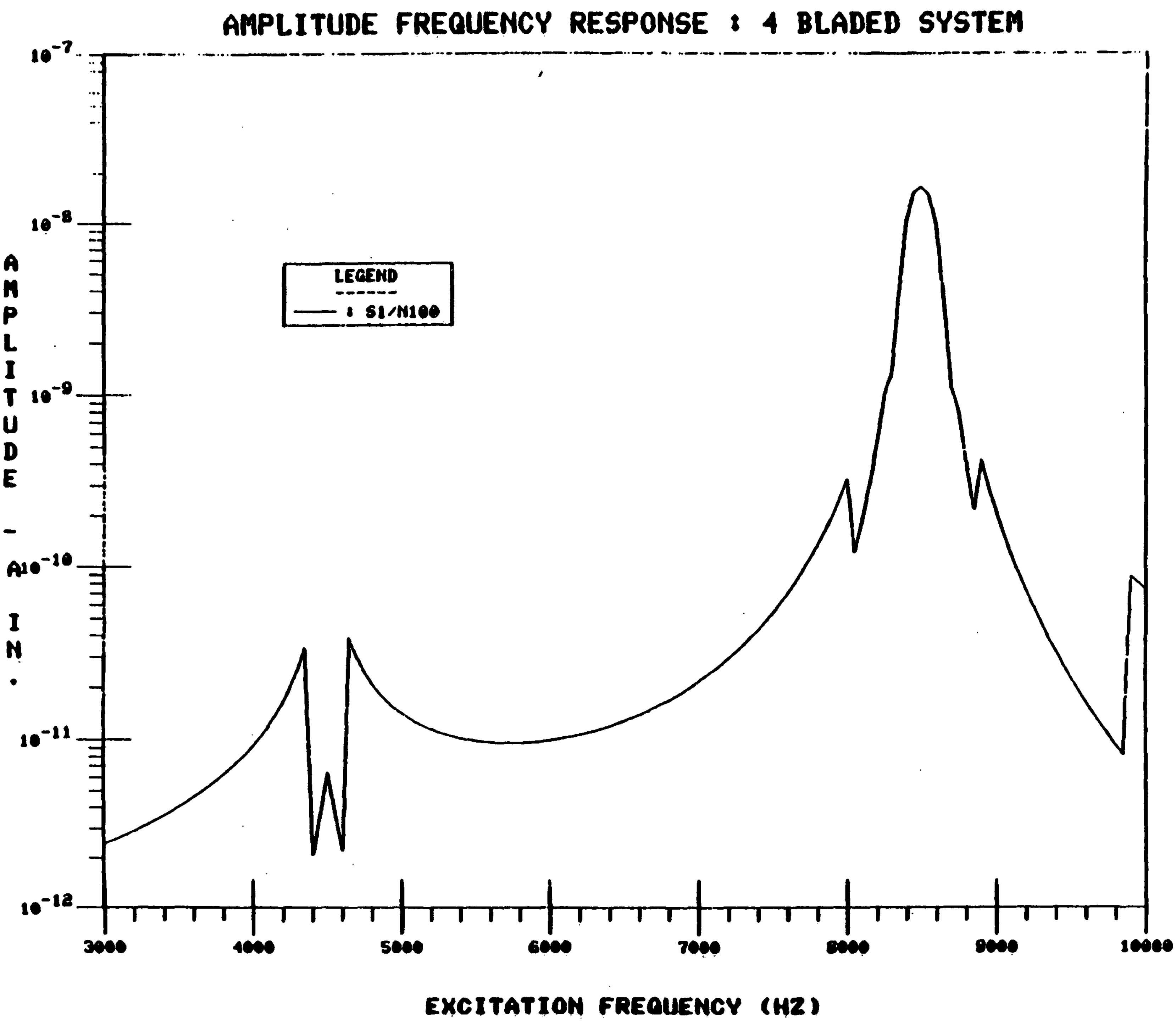


Figure 2. Blade Dynamic Response.

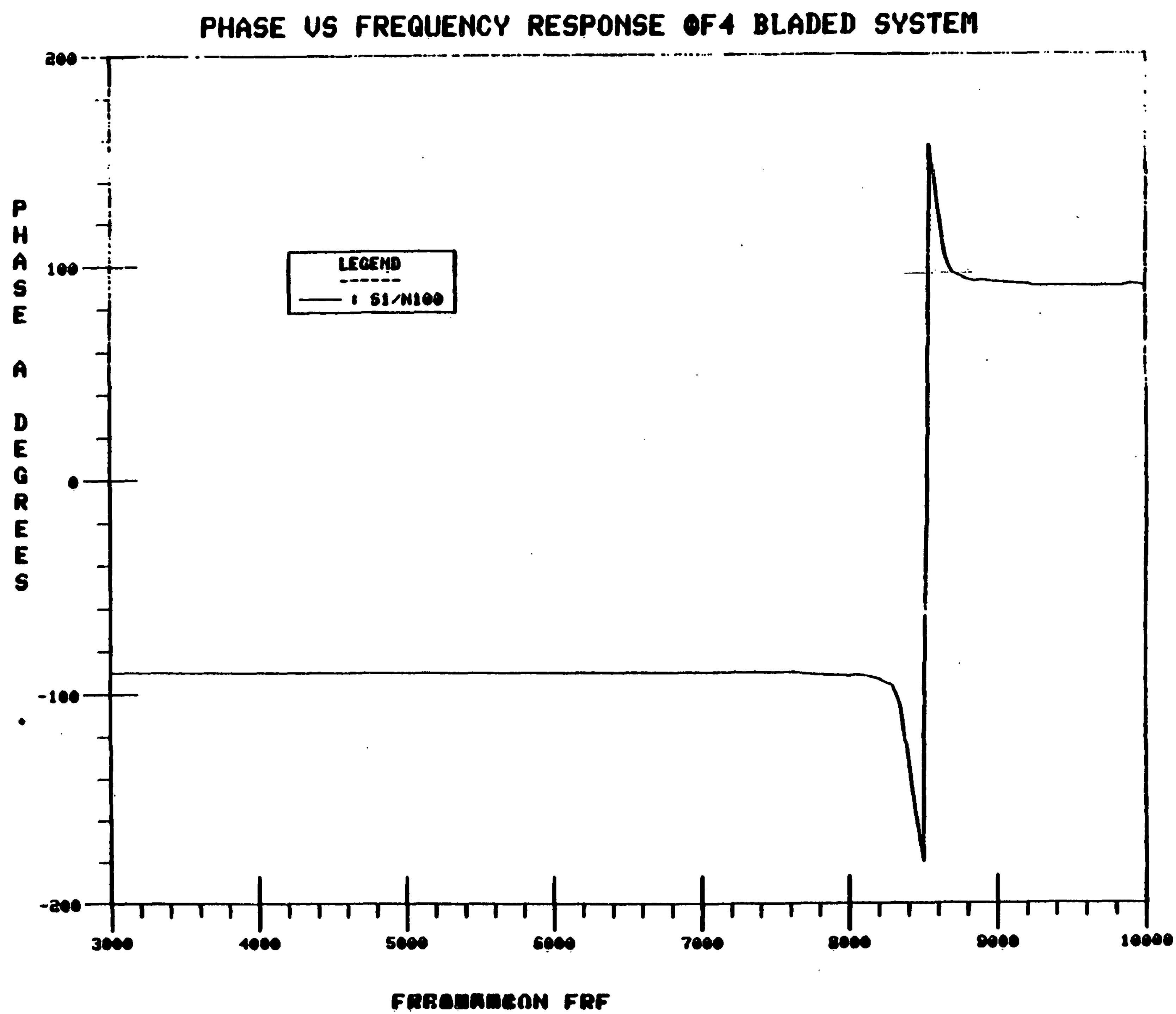


Figure 2. Blade Dynamic Response (Continued).

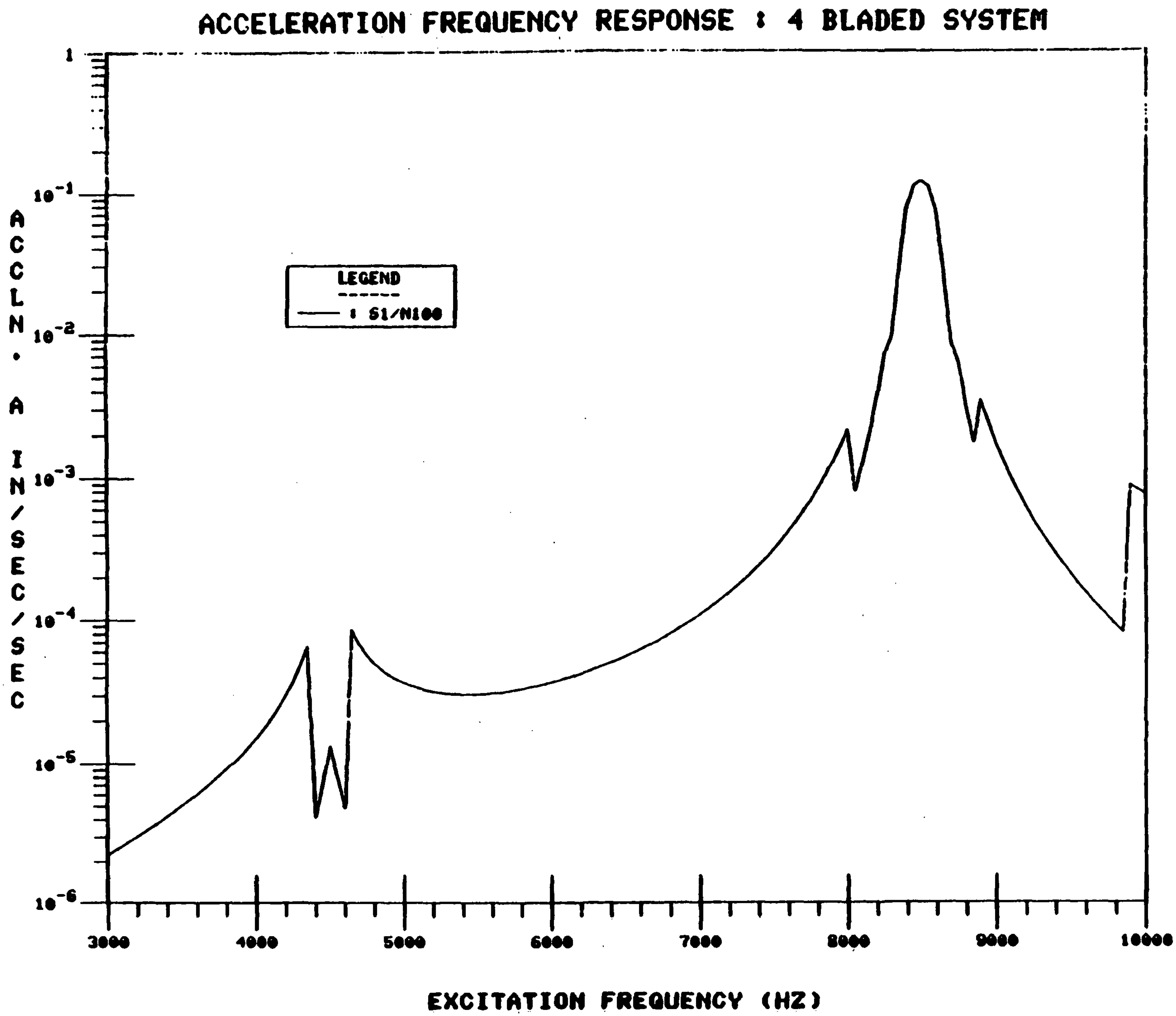


Figure 2. Blade Dynamic Response (Continued).

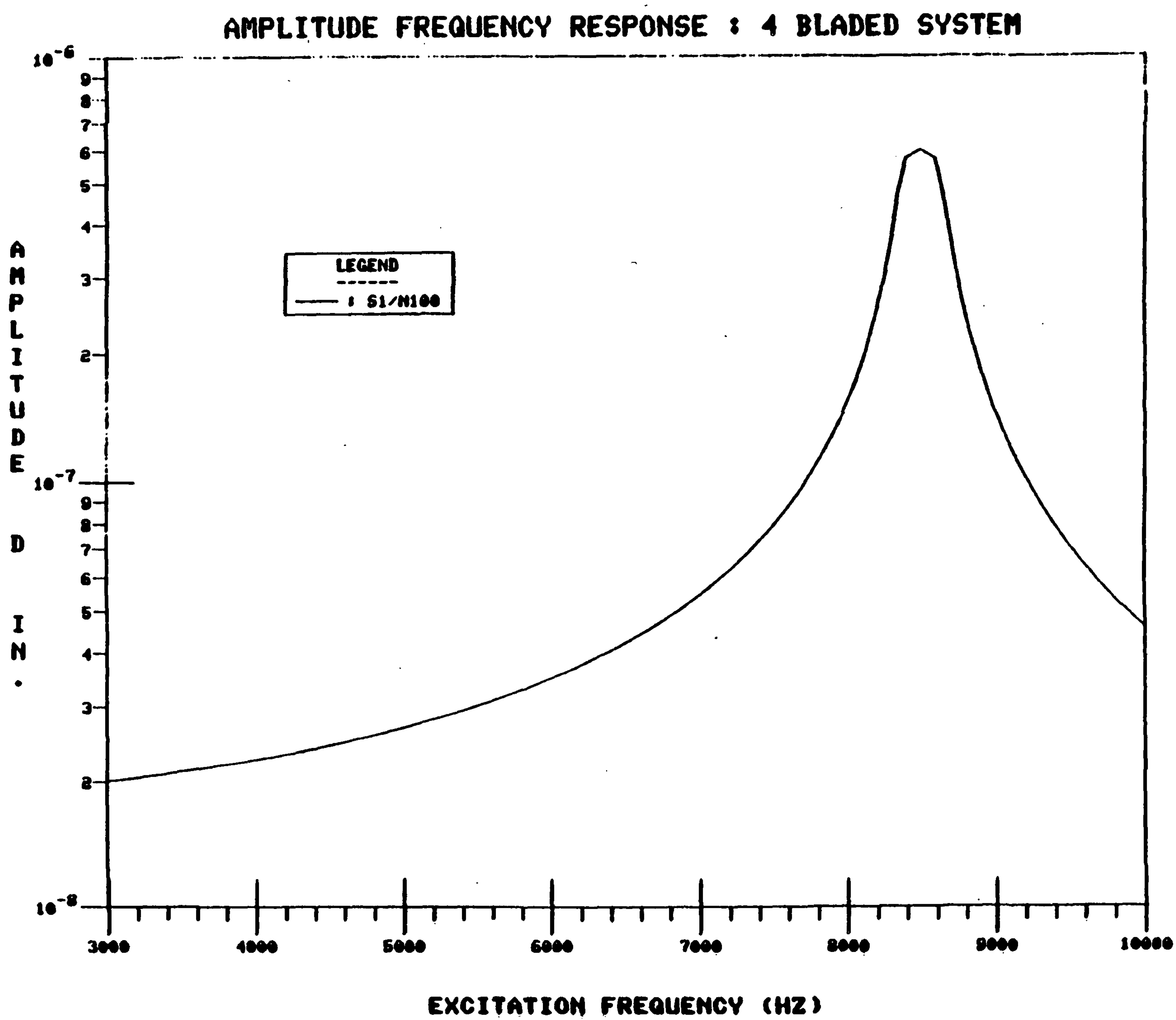


Figure 2. Blade Dynamic Response (Continued).

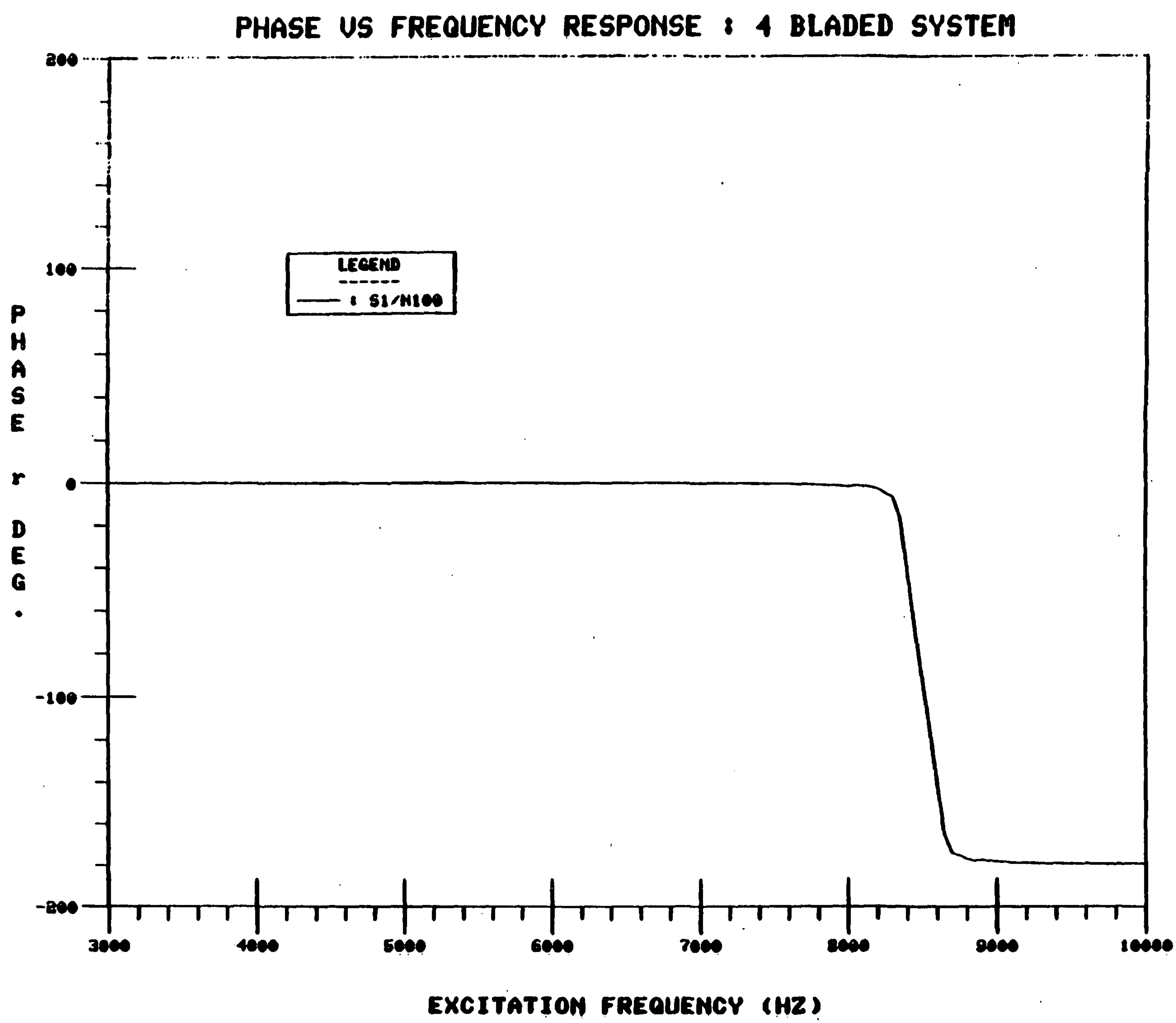


Figure 2. Blade Dynamic Response (Continued).

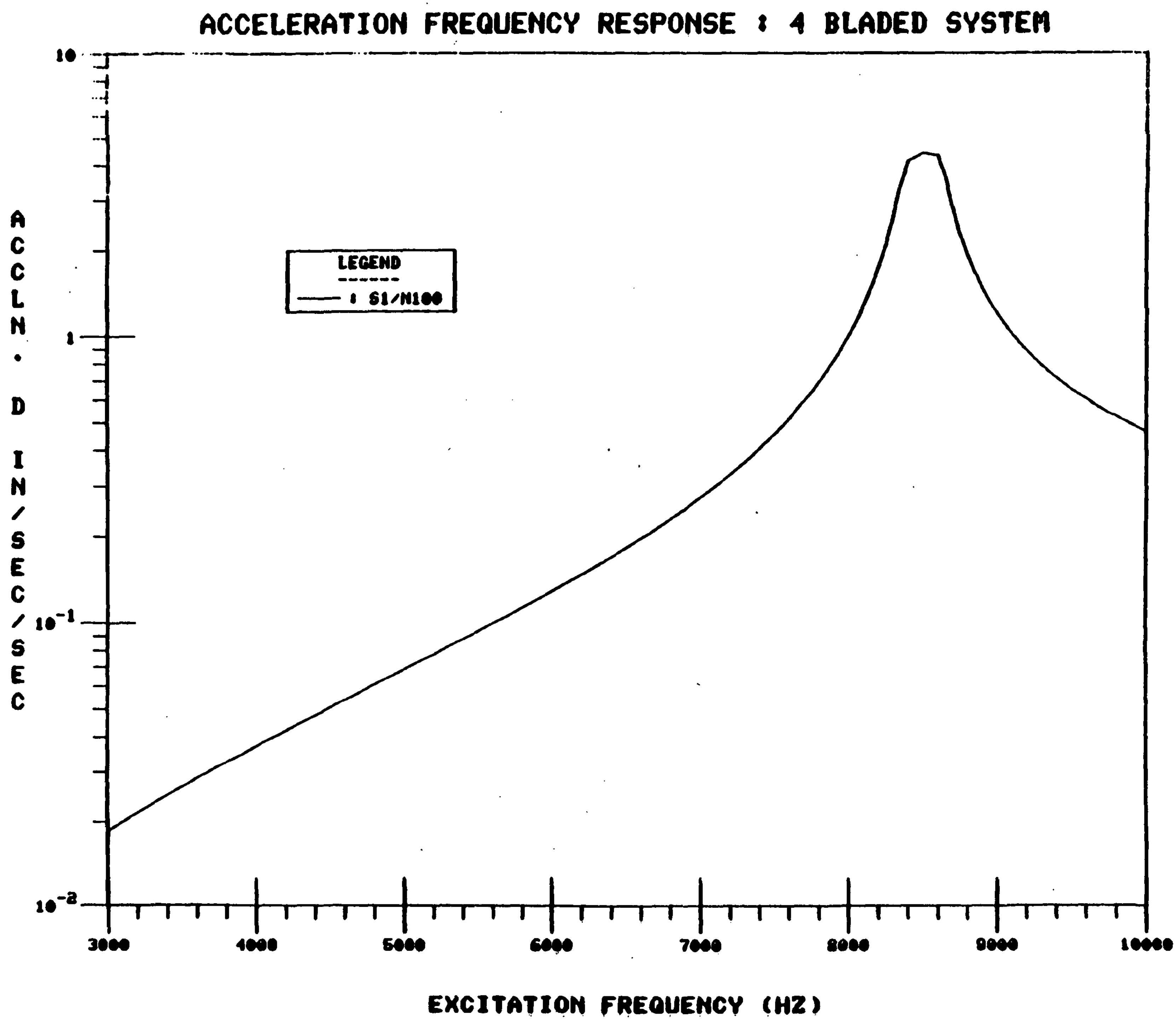


Figure 2. Blade Dynamic Response (Concluded).

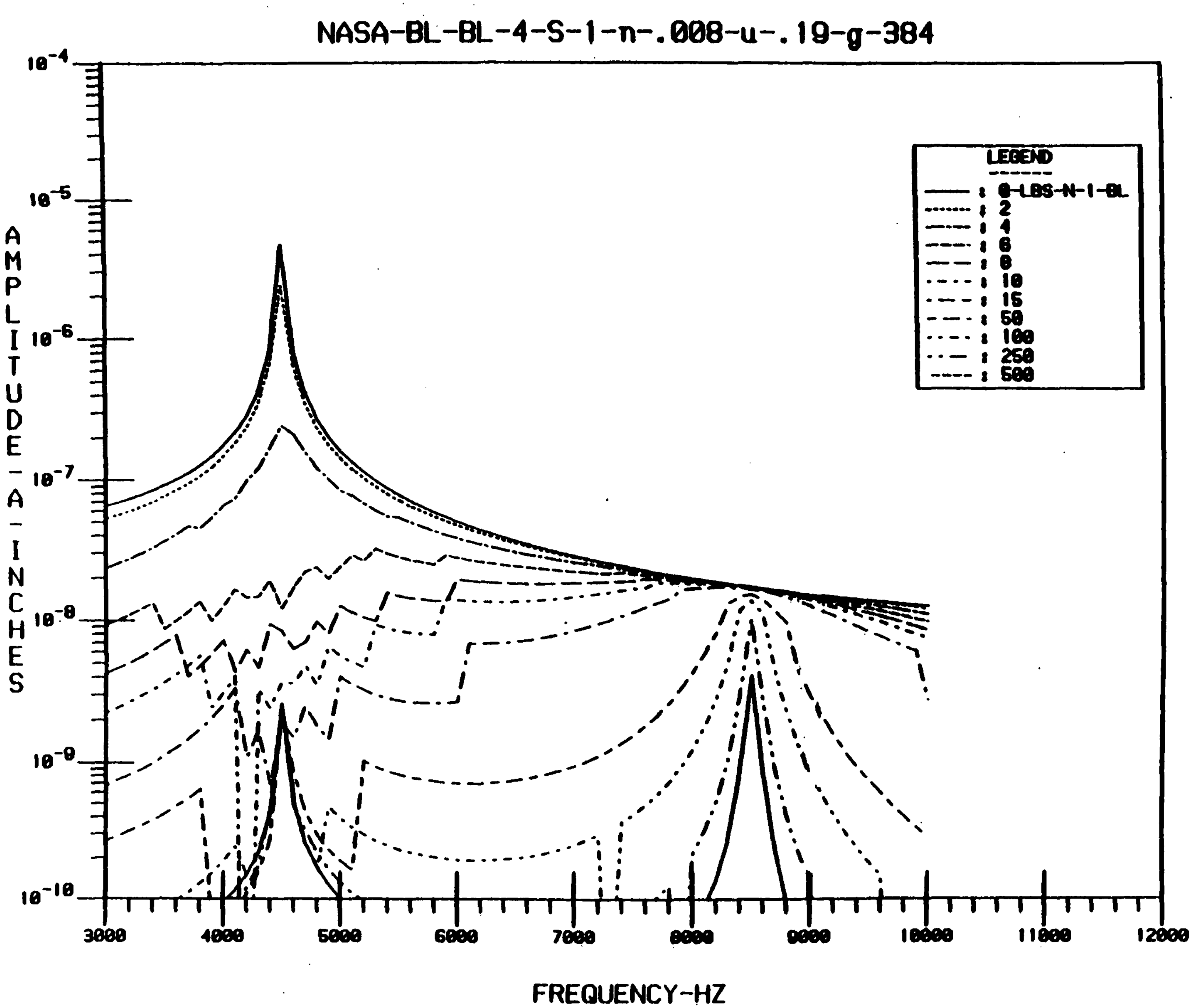


Figure 3. Amplitude Versus Frequency Response for Various Values of N_{ν}.

APPENDIX A

INPUT DATA FORMAT

1. First column in each line should be blank.

2. All numbers are read with a "list-directed" format: (READ(9,*) ABC).

   a. Numbers must be placed in the correct order
   b. Numbers read <u>within a particular group</u> may be separated by:
      i. one or more spaces and/or
      ii. one comma and/or
      iii. may be on the following line
   c. Numbers may be written in I,E,F,etc. formats.
   d. The first number of a new group must appear on a new line.

3. Data groups must be in the order specified.

GROUP	VARIABLES	FORMAT AS READ (*)=List directed	NOTES
1	TITLE - A problem title	1x,A5Ø	
2	NL - Number of blades	(*)	
3	OMEGAØ - Initial frequency (Hz) OMGEND - Final frequency DOMEGA - Step size	(*)	
4	ITMIN - Minimum iterations ITMAX - Maximum iterations EPS - Error tolerance EPS1 - Error tolerance ZLIM - Error tolerance	(*) - - - - - .... - - - - -	Set=2 is OK Set=50 is OK Set=10^{-8} is OK Set=10^{-4} is OK Set=10^{-12} is OK
5	TLIM - Program run-time limit (sec.)	(*) - - - - -	Set=200 is OK
6	ETA1 - See diagram ETA2 ETA3 MU	(*)	See Figure 1
7	GRAVIT		Gravitational Constant
8-15	See next page		
8	M1		
9	M2 - See diagram		
10	K1		
11	K2		
12	K3		
13	S		
14	DELTA		
15	NN		
16	N		

c. Data (continued)

Groups 8-16 are all identified in a similar manner. Each group has a single variable which must be specified for <u>each blade</u>. There are two methods of specifying the variable:

1. If all values are the same:

   a. Line 1: Any group of letters or numbers which contain the word "ALL." Format (1x,A2Ø).

   b. Line 2: The value. Format (*).

2. If all values are <u>not</u> the same:

   a. Line 1: Anything. This line will be read, then ignored.

   b. Line 2-(··): The correct values in order of baldes: 1,2,. . .,etc. Format (*).

For each group, the program reads the first line. If it finds the word "ALL," it goes to the next line and reads the single value. If "ALL" is not found, then it reads a value for each blade, starting at the next line down.

NOTE: The units of spring stiffness, mass, gravitational constant, and frequency must be consistent.

APPENDIX B

NOTES ON RUNNING BPLOT

1. Input Files

BPLOT must have at least one input data file to run. It can handle up to 20 input data files. Each file has one x-value column and up to 50 y-value columns. The program BLADE generated file has frequencies in the x-value column and blade response in y-value columns, one column for each blade. The file is in the following format:

Line	Variables	Format
1	Title	A5Ø
2	Number of y columns	I1Ø
3-on	Point no., x-value, y-values	I1Ø,(**)(1PE14.5)
Last line	-999, any values	I1Ø,(**)(1PE14.5)

2. Output Curves

BPLOT will output up to 50 curves per data file, as specified by the user. The maximum number of curves on a single plot is then 1,000 curves, that is, 20 data files with 50 curves each.

The program has ten types of drawn lines: dotted, solid, dashed, etc. These are indexed to the order in which the user inputs data and are not chosen directly by the user. If more than ten curves are drawn on a single plot, the cycle is begun again in the program.

3. Axis Limits

Limits are chosen by the user but may be rounded in the program to different values. For example, if the user specifies an x-axis with units to begin at 0.0 and end at 9.7, the program will plot an axis from 0.0 to 10.0.

Special attention should be given to limits specified on log axes. In general, these limits will be rounded to the next exact power of ten which still includes all the data requested on the plot. Occasionally, BPLOT will round

value which was input as an exact power of ten to the next power. When this happens, input a value slightly different from the desired value.

4. Labels, Title, Legend

The user can input axis labels, a plot title, and a legend which briefly identifies each curve. To not display a label or title, enter several spaces at the appropriate prompt and return.

The legend is optional. It is plotted in a box when the curves have been drawn. The user places the box on the graph by setting the point for the upper left corner of the box. This point is set by moving the cross-hair or the cursor (depending upon the type of the terminal) to the desired point, followed by typing a single character (any character).

Maximum number of characters allowed are

Labels - 30
Title - 50
Legend - 20 per curve.

5. Ending the Plot

The program will pause for the user to make a hard copy if desired. To continue, type a single character. A prompt will appear - "SAME PLOT AGAIN? (Y/N)...:". An affirmative response "Y" will result in redrawing of the same plot. This allows the user to replace the legend box if necessary. A negative response "N" to the prompt allows a user to specify new data files, new plots, etc.

6. All alphanumeric response should be entered in upper case characters only. That is, for a "yes" response, the user should enter "Y", not "y".

7. Terminal Baud Rate

The program prompts the user to enter baud rate ("ENTER CHAR PER SECOND"). This is the terminal baud rate

divided by 10. Generally it will be 960. Choices are 30, 120, 480, and 960.

8. Terminal Type

This is the Tektronix terminal type being used. Enter the terminal model number, e.g., 4052, 4014, etc. in response to the prompt.

9. Display Output Summary

A "yes" response to the prompt causes the display covering all points if this is the first run-through. Otherwise it covers only the points in the requested x-range. The "OUTPUT SUMMARY" prompt appears for each file connected.

10. Plot Grid

Grid is linear on both axes unless otherwise changed.

APPENDIX C

PROGRAM LISTINGS

PROGRAM DATAFORM.FOR

```
      PROGRAM MAIN
C
C !-----------------------------------------!
C !     DATA FILE FORMATTER                 !
C !                                         !
C !  1. WRITES AN INPUT DATA FILE           !
C !                                         !
C !  2. INTERACTIVE PROCEDURE ONLY          !
C !                                         !
C !  3. WRITTEN FOR FORTRAN/77 COMPILER     !
C !                                         !
C !  4. OUTPUT FILE SPECS:                  !
C !      NAME   : DFF                       !
C !      ACCESS : SEQUENTIAL                !
C !      FORM   : FORMATTED                 !
C !      RECL   : 80                        !
C !                                         !
C !-----------------------------------------!
C
      CHARACTER*50 TITLE
      CHARACTER*20 ST(16), FNAME
      CHARACTER*1  ANS
C
      DIMENSION MODTAB (20)
C
      COMMON /BLK1/ A(9,50), NUM(9), NL, R9, I9
      COMMON /BLK2/ ST
C
      REAL MU
C
      DO 4  J=1,16
    4 MODTAB (J) = 1
C
      R9 = -999.
      I9 = -999
C
      ISTART = 8
C
    5 WRITE (6, 605)
C
      ST(1)  = 'TITLE'
      ST(2)  = 'NO.  OF BLADES'
      ST(3)  = 'OMEGAO,OMGEND,DOMEGA'
      ST(4)  = 'IMN,IMX,EPS,EPS1,ZLM'
      ST(5)  = 'TIME LIMIT'
C
      ST(6)  = 'ETA1,ETA2,ETA3,MU'
      ST(7)  = 'GRAVITATIONAL CONST. '
      ST(8)  = 'M1'
      ST(9)  = 'M2'
C
      ST(10) = 'K1'
      ST(11) = 'K2'
      ST(12) = 'K3'
      ST(13) = 'S'
      ST(14) = 'DELTA'
C
      ST(15) = 'NN'
      ST(16) = 'N'
C
      MOD = 0
C
```

```
C------TITLE MODEL.
C
   10 WRITE (6, 610)
      READ (5, 510, ERR=10) TITLE
      MOD = 1
      IFLAG = 1
      IF (MODTAB(MOD) .EQ. 2) IFLAG=2
      CALL CHKC (MOD,MODTAB,IFLAG,TITLE)
      MODTAB(MOD) = 3
      GO TO (20, 200, 250), IFLAG
C
C------NO. OF BLADES.
C
   20 WRITE (6, 620)
      READ (5, *, ERR=20) NL
      MOD = 2
      IFLAG = 1
      IF (MODTAB(MOD) .EQ. 2) IFLAG=2
      CALL CHKI (MOD,MODTAB,IFLAG,NL)
      MODTAB(MOD) = 3
C
      IF ( ((NL .LT.     1) .OR.
     +       (NL .GT.    50))  .AND.
     +      ( NL .NE.  -999)) THEN
   25   WRITE (6, 625)
        GO TO 20
      END IF
C
      GO TO (30, 200, 250) , IFLAG
C
C------OMEGA INPUT.
C
   30 WRITE (6, 630)
      READ (5, *, ERR=30) OMEGAO, OMGEND, DOMEGA
      MOD = 3
      IFLAG = 1
      IF (MODTAB(MOD) .EQ. 2) IFLAG=2
      CALL CHKR (MOD,MODTAB,IFLAG,OMEGAO)
      CALL CHKR (MOD,MODTAB,IFLAG,OMGEND)
      CALL CHKR (MOD,MODTAB,IFLAG,DOMEGA)
      MODTAB(MOD) = 3
      GO TO (40, 200, 250) , IFLAG
C
C------ITERATION PARAMETERS.
C
   40 WRITE (6, 640)
      READ (5, *, ERR=40) ITMIN, ITMAX, EPS, EPS1, ZLIM
      MOD = 4
      IFLAG = 1
      IF (MODTAB(MOD) .EQ. 2) IFLAG=2
      CALL CHKI (MOD,MODTAB,IFLAG,ITMIN)
      CALL CHKI (MOD,MODTAB,IFLAG,ITMAX)
      CALL CHKR (MOD,MODTAB,IFLAG,EPS   )
      CALL CHKR (MOD,MODTAB,IFLAG,EPS1  )
      CALL CHKR (MOD,MODTAB,IFLAG,ZLIM  )
      MODTAB(MOD) = 3
      GO TO (50, 200, 250), IFLAG
C
C------RUN TIME LIMIT (SECS).
C
   50 WRITE (6, 650)
```

```
      READ (5, *, ERR=50) TLIM
      MOD = 5
      IFLAG = 1
      IF (MODTAB(MOD) .EQ. 2) IFLAG=2
      CALL CHKR (MOD,MODTAB,IFLAG,TLIM   )
      MODTAB(MOD) = 3
      GO TO (60, 200, 250), IFLAG
C
C------ETA1 INPUTS.
C
   60 WRITE (6, 660)
      READ (5, *, ERR=60) ETA1, ETA2, ETA3, MU
      MOD = 6
      IFLAG = 1
      IF (MODTAB(MOD) .EQ. 2) IFLAG=2
      CALL CHKR (MOD,MODTAB,IFLAG,ETA1    )
      CALL CHKR (MOD,MODTAB,IFLAG,ETA3   )
      CALL CHKR (MOD,MODTAB,IFLAG,ETA2   )
      CALL CHKR (MOD,MODTAB,IFLAG,MU     )
      MODTAB(MOD) = 3
      GO TO (62, 200, 250), IFLAG
C
C------GRAVITATIONAL CONSTANT.
C
   62 WRITE (6, 662)
      READ (5, *, ERR=62) GRAVC
      MOD = 7
      IFLAG = 1
      IF (MODTAB(MOD) .EQ. 2) IFLAG=2
      CALL CHKR (MOD,MODTAB,IFLAG,GRAVC )
      MODTAB(MOD) = 3
      GO TO (70, 200, 250), IFLAG
C
C------MULTIPLE MODULE INPUT.
C
   70 DO 80  I=ISTART,16
        J=I-7
        MOD = I
        CALL INMAT (J,RIN)
        IFLAG = 1
        IF (MODTAB(MOD) .EQ. 2) IFLAG=2
        CALL CHKR (I   ,MODTAB,IFLAG,RIN    )
        MODTAB(I) = 3
        GO TO (80, 200, 250), IFLAG
C
   80 CONTINUE
C
      MOD = 17
      GO TO 250
C
C------REGULAR INPUT DONE.
C
  200 CONTINUE
C
C------CHECK FOR INTERRUPTS.
C
      DO 210  I=1,16
C
        IF (MODTAB(I) .EQ. 3) GO TO 210
        IF (I .GE. 8) THEN
          ISTART = I
```

```
          GO TO 70
        ELSE
          GO TO (10,20,30,40,50,60,62), I
        END IF
  210 CONTINUE
      MOD = 17
      GO TO 250
C
  250 WRITE (6, 6250)
  260 READ (5, 5260, ERR=250) ANS
C
      MODTAB(MOD) = 2
C
      IF ( (ANS .EQ. 'Y') .OR.
     +     (ANS .EQ. 'y')) GO TO 270
      IF ( (ANS .NE. 'N') .AND.
     +     (ANS .NE. 'n')) GO TO 250
      GO TO 295
C
  270 WRITE (6, 6270)
C
      DO 274  K=1,17
        IF (MODTAB(K) .EQ. 1) GO TO 278
  274 CONTINUE
      K = 17
  278 MP1 = K-1
      DO 280  I=1,MP1
        JSTLEN = INDEX( ST(I), 'ALL' )
        IF (JSTLEN .EQ. 0) JSTLEN=21
        JSTLEN =JSTLEN - 1
  280 WRITE (6, 6280) I, ST(I)(1:JSTLEN)
  285 WRITE (6, 6285)
C
  290 WRITE (6, 6290)
      READ (5, *, ERR=290) INEXT
      IF (INEXT .EQ. 100) GO TO 295
      IF ( (INEXT .LT. 1) .OR.
     1     (INEXT .GT. MP1) ) GO TO 270
C
      IF (INEXT .GE. 8) ISTART=INEXT
      MODTAB (INEXT) = 2
        IF (INEXT .EQ. 2) THEN
          DO 292  J=8,16
  292     MODTAB(J) = 1
        END IF
C
      GO TO (10,20,30,40,50,60,62,70), INEXT
      GO TO 70
C
  295 IF (MOD .GE. 17) GO TO 300
      GO TO 200
C
  300 CONTINUE
C
  310 WRITE (6, 6310)
      FNAME = 'DFF'
      OPEN (    UNIT = 10,
     +          FILE = FNAME,
     +        STATUS = 'NEW',
     +        ACCESS = 'SEQUENTIAL',
     +          FORM = 'FORMATTED'    )
```

```
C
  320 WRITE (10, 6320) TITLE
      WRITE (10, *) NL
      WRITE (10, *) OMEGAO, OMGEND, DOMEGA
C
      WRITE (10, *) ITMIN, ITMAX, EPS, EPS1, ZLIM
      WRITE (10, *) TLIM
      WRITE (10, *) ETA1, ETA2, ETA3, MU
      WRITE (10, *) GRAVC
C
  330 DO 350  J=1,9
         JST = J+7
  340    WRITE (10, 6340) ST(JST)
         L = NUM(J)
         ILINE = L/4
         IREM  = L - (ILINE*4)
         IF (IREM .GT. 0) ILINE=ILINE+1
         N2 = 0
         DO 344  M1=1,ILINE
           N1 = N2+1
           N2 = MIN( (N1+3), L )
  342      WRITE (10,*) (A(J,NX), NX=N1,N2)
  344    CONTINUE
  350 CONTINUE
C
      CLOSE (10)
C
  360 WRITE (6, 6360)
C
      STOP
C
  605 FORMAT ( //10X, 'DATA FILE FORMATTER',
     +             /10X, '-------------------',
     + //2X, 'TO INTERRUPT OR MODIFY, ',
     +  /2X, 'ENTER -999 FOR ANY INPUT VALUE. ' ,/)
C
  610 FORMAT ( /2X, 'ENTER TITLE (50 CHAR MAX).......:  '$ )
  620 FORMAT ( /2X, 'ENTER NUMBER OF BLADES..........:  '$ )
C
  625 FORMAT (
     +  /5X, '**           INPUT ERROR :             **',
     +  /5X, '** MIN. ALLOWED NO. OF BLADES =  1 **',
     +  /5X, '** MAX. ALLOWED NO. OF BLADES = 50 **', /)
C
  630 FORMAT ( /2X, 'ENTER OMEGAO, OMGEND, DOMEGA....:  '$ )
  640 FORMAT ( /2X, 'ENTER ITMIN, ITMAX, EPS, EPS1, '
     1         /2X, 'ZLIM............................:  '$ )
C
  650 FORMAT ( /2X, 'ENTER PROGRAM RUN-TIME LIMIT....:  '$ )
  660 FORMAT ( /2X, 'ENTER ETA1, ETA2, ETA3, MU.......:  '$ )
C
  662 FORMAT ( /2X, 'ENTER GRAVITATIONAL CONSTANT....:  '$ )
C
 6250 FORMAT ( /2X, 'ANY MODIFICATIONS ?  (Y/N) .....:  '$ )
 6270 FORMAT ( /2X, 'MODULES AVAILABLE FOR CHANGES :'     )
C
 6280 FORMAT ( 8X, I2, '. ', A )
 6285 FORMAT ( 7X, '100. NO CHANGES'    )
 6290 FORMAT ( /2X, 'ENTER MODULE NUMBER FROM ABOVE..:  ' $)
C
 6310 FORMAT ( /2X, 'WRITING TO FILE...'                   )
```

```
C
 6320 FORMAT ( 1X, A50 )
 6340 FORMAT ( 1X, A20 )
 6345 FORMAT ( 20 (2X, 4(1PE18.10), /)  )
C
 6360 FORMAT ( /2X, 'DATA FORMAT PROGRAM IS DONE.', ///)
C
  510 FORMAT ( A50 )
 5260 FORMAT ( A1  )
 5300 FORMAT ( A20 )
C
      END
C
      SUBROUTINE INMAT (IND,ROUT)
C
      CHARACTER*20 ST(16)
      CHARACTER*4  C9
      CHARACTER*4  ANS
C
      COMMON /BLK1/ A(9,50), NUM(9), NL, R9, I9
      COMMON /BLK2/ ST
C
      IN6 = IND+7
      C9  = '-999'
C
  100 WRITE (6, 6100) ST(IN6) (1:5)
      READ (5, 5100, ERR=100) ANS
      IF ( (ANS .NE. 'Y') .AND.
     +     (ANS .NE. 'y') .AND.
     +     (ANS .NE. 'N') .AND.
     +     (ANS .NE. 'n')) GO TO 100
      ST(IN6)(9:11) = '   '
C
      L = NL
      IF ( (ANS .EQ. 'Y   ') .OR.
     1     (ANS .EQ. 'y   '))  THEN
        L=1
        ST(IN6) (9:11) = 'ALL'
      END IF
C
      IF (ANS .EQ. C9) THEN
        ROUT = R9
        RETURN
      END IF
C
      DO 120  J=1,L
  110   WRITE (6, 6110) ST(IN6) (1:5), J
        READ (5, *, ERR=110) ROUT
        A(IND,J) = ROUT
        IF (ROUT .EQ. R9) RETURN
  120 CONTINUE
C
      NUM(IND) = L
      RETURN
C
 5100 FORMAT ( A4 )
 6100 FORMAT ( /2X, 'FOR ', A5, ' : ARE ALL VALUES',
     1              ' EQUAL ? (Y/N)..: '$ )
C
 6110 FORMAT ( /2X, 'ENTER ', A5, '(', I2, ').....',
     1              '....: '$                   )
```

```
C
      END
C
      SUBROUTINE CHKC (MOD,MODTAB,IFLAG,VAL)
C
      CHARACTER*(*) VAL
      DIMENSION MODTAB(*)
C
      IF (IFLAG .EQ. 3) RETURN
      IF (VAL .EQ. '-999') THEN
        MODTAB (MOD) = 2
        IFLAG        = 3
      END IF
C
      RETURN
      END
C
      SUBROUTINE CHKI (MOD,MODTAB,IFLAG,VAL)
C
      INTEGER VAL
      DIMENSION MODTAB(*)
C
      IF (IFLAG .EQ. 3) RETURN
      IF (VAL .EQ. -999) THEN
        MODTAB (MOD) = 2
        IFLAG        = 3
      END IF
C
      RETURN
      END
C
      SUBROUTINE CHKR (MOD,MODTAB,IFLAG,VAL)
C
      DIMENSION MODTAB(*)
C
      IF (IFLAG .EQ. 3) RETURN
      IF (VAL .EQ. -999. ) THEN
        MODTAB (MOD) = 2
        IFLAG        = 3
      END IF
C
      RETURN
      END
```

RESPONSE ANALYSIS PROGRAM BLADE.FOR

```
      PROGRAM BLADE
C
C ***********************************************************
C *                                                         *
C *        BLADE DYNAMICS PROGRAM                           *
C *                                                         *
C *---MODIFIED TO OUTPUT ACCELERATIONS - 2/15/83 TH-*
C *                                                         *
C *---MODIFIED TO USE ETA2 - ML SONI/ T HELD 8/22/83*
C *                                                         *
C *---MODIFIED TO CLEAN UP DATA I/O     - 5/08/84 TH-*
C *                                                         *
C ***********************************************************
C
C
C-----------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
C-----------------------------------------------------------
      DIMENSION K(3,50), M1(50), M2(50), N(50), NN(50)
      DIMENSION X1(50), X2(50), S(50), DELTA(50)
      DIMENSION DFACT(50)
      DIMENSION A(50), ALPHA(50), D(50), GAMMA(50)
C
      DIMENSION PL(50,2), G(50), W(52), V(52)
      DIMENSION Q(100), Z(102), R(2,50), ABA(52)
      DIMENSION Z1(100), IND(50)
C
      DIMENSION P(5050), DET(5350), ICHNG(200)
      DIMENSION ZINIT(50), INDQ(50), AL(50)
      DIMENSION ALPOUT(50), GAMOUT(50), AACC(50), DACC(50)
C
      CHARACTER*82 HEADER
      CHARACTER*50 TITLE, BLNK50
      CHARACTER*25 BDP
      CHARACTER*6 AC
      CHARACTER*3 PLR
      CHARACTER*1  BLNK1
C
      COMMON /BLK1/ NL, M1, K, M2, N, S, DELTA, NN, OMEGAO,
     1              DOMEGA, OMGEND, ETA1, ETA3, AL, MU, AOMEGA,
     2              GRAVIT, DFACT, IDELTA, ETA2
C
      COMMON /BLK2/ HEADER, TITLE, BLNK50, BDP, BLNK1, AC
      COMMON /BLK3/ IPAGE, ISTEP, OMOUT, ITNO, DIFF, DIFF1
C
      COMMON /BLK4/ ITMIN, ITMAX, EPS, EPS1, TLIM, ZLIM
      COMMON /PARS/ IREAD,IKM,ISD
C
C
      REAL*8   M1, M2, K, NN, MU, N
      REAL*4   PROGTM
C
      LOGICAL KEY
      LOGICAL NEW
      LOGICAL SW
      LOGICAL TEST, TEST1
C
      DATA PI/3.1415926535898/
C
      IBPOST = 1
C
C---------------------------
```

```
C     FOR CDC, USE OPEN STATEMENTS
C     FOR INPUT AND OUTPUT ON UNITS 5
C     AND 6.
C
       OPEN (9, STATUS='OLD')
       IF (IBPOST .EQ. 1) THEN
         OPEN (51, STATUS='NEW', RECL=724,
     1        RECORDTYPE='VARIABLE', FILE='AXXOUT')
         OPEN (52, STATUS='NEW', RECL=724,
     1        RECORDTYPE='VARIABLE', FILE='DXXOUT')
         OPEN (53, STATUS='NEW', RECL=724,
     1        RECORDTYPE='VARIABLE', FILE='ALPOUT')
         OPEN (54, STATUS='NEW', RECL=724,
     1        RECORDTYPE='VARIABLE', FILE='GAMOUT')
         OPEN (55, STATUS='NEW', RECL=724,
     1        RECORDTYPE='VARIABLE', FILE='AACCEL')
         OPEN (56, STATUS='NEW', RECL=724,
     1        RECORDTYPE='VARIABLE', FILE='DACCEL')
       END IF
C---------------------------
C
      KEY    = .FALSE.
C
      NEW = .FALSE.
      SW  = .FALSE.
C
C------------------------------------
C       USE 'SECNDS' FOR VAX TIMER.
C       USE 'SECOND' FOR CDC TIMER.
C
      PROGTM = SECNDS(0.0)
C------------------------------------
      REWIND 9
      CALL INDATA
C
C-----WRITE BPOST INITS------------
      IF (IBPOST .NE. 1) GO TO 512
  510   WRITE (51, 6510) TITLE
        WRITE (52, 6510) TITLE
        WRITE (53, 6510) TITLE
        WRITE (54, 6510) TITLE
        WRITE (55, 6510) TITLE
        WRITE (56, 6510) TITLE
 6510   FORMAT (A50)
C
  511 WRITE (51, 6511) NL
      WRITE (52, 6511) NL
      WRITE (53, 6511) NL
      WRITE (54, 6511) NL
      WRITE (55, 6511) NL
      WRITE (56, 6511) NL
C
 6511 FORMAT (I10)
  512 CONTINUE
C
C......  SET OUTPUT FORMATTING CONSTANTS.
C
      LMAX = 60
      LHED = 2
C
      IPERS = (LMAX-LHED) / (2*NL +7)
```

```
      JPERS = LMAX - LHED - 7
C
C......   END SET
C
    6 CONTINUE
C
C
C......   BEGIN LOOP STEPPING OMEGA...
C
C          ...INITIALIZE ARRAYS...
C
      OMEGA = OMEGAO-DOMEGA
      ISTEP = 0
    1 CONTINUE
C
      OMEGA = OMEGA + DOMEGA
      ISTEP = ISTEP + 1
      OMHZ  = OMEGA / (2.*PI)
      IF (OMEGA .GT. (OMGEND+DOMEGA+1.)) GO TO 100
C
C......   SET ACCELERATION MULT. FACTOR
C
      GFAC1 = (OMEGA*OMEGA)/GRAVIT
C
C......   SET UP FOR VARIABLE DELTA'S
C
C         IDELTA .EQ. 1 : DELTA VARIES WITH FREQUENCY
C                .NE. 1 : DELTA CONSTANT FOR EACH BLADE
C
      IF (IDELTA .EQ. 1) THEN
        DO 802  II=1,NL
          IF ( ABS(DFACT(II)) .LT. 1.E-15 ) THEN
            DELTA(II) = 0.0
          ELSE
            DELTA(II) = OMEGA / (DFACT(II) * FLOAT(NL))
          END IF
  802   CONTINUE
      END IF
C
C------------------------------------------------------------------
      ELPSTM = SECNDS(PROGTM)
C     ELPSTM = SECOND()
      IF (ELPSTM .GT. TLIM) THEN
  800   WRITE (6,6800) ELPSTM
 6800 FORMAT (//, 10X, 'PROGRAM TIME LIMIT EXCEEDED.',
     1      //, 10X, 'FINAL TIME (SEC) = ', F10.2, ////)
        GO TO 518
      END IF
C------------------------------------------------------------------
C
  702 FORMAT (E20.10)
      OMOUT = OMEGA/(2.*PI)
C
      SN     = -1
      SN     = -0.5
      NX     =  0
      NX     = NX + 1
      NTIME  =  1
      TO     =  1.E-5
C
      NXLIM = 10
```

```
      NXLIM = 20
C
      IF (NEW) GO TO 9
      DO 8  I=1,NL
        W(I) = 0.
        V(I) = 0.
    8 CONTINUE
C
    9 CONTINUE
C
      DO 10  NU=1,NL
        NUM1 = NU-1
        IF (NU .EQ.  1) NUM1=NL
C
C        ...CALCULATE 'P' TERMS...
C
C
      PL(NU,1) = K(2,NU) + K(3,NU) + K(3,NUM1) - M2(NU)*OMEGA**2
     1           - K(1,NU) * M1(NU) * OMEGA**2*((1.+ETA1**2) *
     2           K(1,NU)-M1(NU) * OMEGA**2) /
     3           ((K(1,NU) - M1(NU) * OMEGA**2)**2 + (K(1,NU)*ETA1)**2)
C
      PL(NU,2) = ETA2 * K(2,NU) + ETA3 * (K(3,NU)+K(3,NUM1)) + ETA1*
     1           K(1,NU) * M1(NU)**2*OMEGA**4/((K(1,NU)-M1(NU)*OMEGA**2)
     2           **2+(K(1,NU)*ETA1)**2)
C
C          ...  CALCULATE  R  TERMS ...
C
      R(1,NU) = (1.+ETA1**2-M1(NU) / K(1,NU)*OMEGA**2) /
     1          ((1.-M1(NU)/K(1,NU)*OMEGA**2)**2 + ETA1**2)
C
      R(2,NU) = (M1(NU)/K(1,NU)*ETA1 * OMEGA**2) /
     1          ((1.-M1(NU)/K(1,NU)*OMEGA**2) **2 + ETA1**2)
C
   10 CONTINUE
C
      ITNO = 0
   11 CONTINUE
      ITNO  = ITNO + 1
      TEST  = .FALSE.
      TEST1 = .FALSE.
      IF (ITNO .GT.  ITMAX) GO TO 56
C
      DO 15  NU=1,NL
        DNU = DELTA(NU)
        IND2 = 2*NU
        IND1 = IND2 - 1
C
        Q(IND1) = S(NU) * (R(1,NU)*COS(DNU)
     1             + R(2,NU)*SIN(DNU))
        Q(IND2) = S(NU) * (R(1,NU)*SIN(DNU)
     1             - R(2,NU)*COS(DNU))
   15 CONTINUE
C
      ML = 2*NL
      MLIM = ML*(ML+1)/2
C
C       USE THE P MATRIX FROM PREVIOUS ITERATION (FREQUENCY)
C
C        ...SET 'P' TO ZERO...
C
```

```
      DO 20  I=1,MLIM
        P(I)=0.
   20 CONTINUE
C
C        ...CONSTRUCT 'P' MATRIX...
C
      P(1) =  PL(1,1)
      P(2) =  PL(1,2) + W(1) + W(NL) + V(1)
      P(3) = -PL(1,1)
C
      DO 12  I=1,NL
        IND(I)    = 0
        INDG(I)   = 0
C----ALPHA SET = 0.  - IF NOT DEFND, KEPT=0.---
        ALPHA(I) = 0.
   12 CONTINUE
C
      DO 30  I=2,NL
        I1 = I*(2*I - 1)
        I2 = I*(2*I + 1)
C
        P(I1)    =  PL(I,1)
        P(I1-1) =  (-ETA3*K(3,I-1)) - W(I-1)
        P(I1-2) = -K(3,I-1)
C
        P(I2)    = -PL(I,1)
        P(I2-1) =  PL(I,2) + W(I) + W(I-1) + V(I)
        P(I2-2) =  K(3,I-1)
        P(I2-3) =  (-ETA3*K(3,I-1)) - W(I-1)
   30 CONTINUE
C
      I3 = (NL-1)*(2*NL-1) + 1
      I4 = NL*(2*NL-1) + 1
C
      P(I3)    = -K(3,NL)
      P(I3+1) =  (-ETA3 * K(3,NL)) - W(NL)
      P(I4)    =  (-ETA3 * K(3,NL)) - W(NL)
      P(I4+1) =  K(3,NL)
C
C        ...NEW 'P' IS COMPLETE...
C
 278  CONTINUE
  601 FORMAT (' ', 11E12.6)
      IJOB = 0
C
C
C     IF (ITNO .EQ. 1) CALL PAGER
C
   91 PLR = '  P'
C     CALL MATOUT (P, PLR, MLIM, ITNO)
C
   92 PLR = ' RH'
C     CALL MATOUT (Q, PLR, ML, ITNO)
C
C
       CALL LEQ2S (P,ML,Q,1,ML,IJOB,ICHNG,DET,IER)
   93 PLR = ' LH'
C     CALL MATOUT (Q, PLR, ML, ITNO)
      CALL QFIX (Q,ML)
C
      IF (IER .NE. 0) THEN
```

```
        CALL PAGER
        WRITE (6, 603) ISTEP, OMHZ, ITNO, IER
      END IF
C
  603 FORMAT (///10X,
     1       '***** WARNING FROM MAIN PROGRAM *****',
     2 /10X, '*****     IMSL ERROR RETURNED     *****',
     3 /10X, '*****     STEP NUMBER : ', I7,   '   *****',
     4 /10X, '*****     OMEGA : ', 1PE13.5,    '   *****',
     5 /10X, '*****     ITERATION   : ', I7,   '   *****',
     6 /10X, '*****     IER         : ', I7,   '   *****',  /)
C
      IF (.NOT.NEW) GO TO 35
      IF ( (IER .EQ. 0) .AND. (ITNO .EQ. 1) )
     1     CALL LOAD (G,ZINIT,ML,1.,0,0.)
C
      IF (IER .EQ. 0) GO TO 35
      IF (NX .LT. NXLIM) GO TO 33
C
      IF (NTIME .EQ. 2) GO TO 1
      IF (NTIME .EQ. 2) STOP
C
      NTIME = 2
      NX = 0
      SN = 1.
      SN = 0.5
C
   33 NX = NX + 1
      T = 0.
      IF (ITNO .EQ. 1) T = TO
C
      CALL LOAD(ZINIT,G,ML,SN,NX,T)
C
      ITNO = 1
C
   35 CONTINUE
C
      DO 40  NU=1,ML
   40 Z(NU) = G(NU)
C
  302 CONTINUE
C
      Z(ML+1) = Z(1)
      Z(ML+2) = Z(2)
C
      DO 45  NU=1,NL
       ABA(NU) = SQRT(Z(2*NU-1)**2 + Z(2*NU)**2)
C
       G(NU) = SQRT( (Z(2*NU)-Z(2*NU+2))**2
     1               + (Z(2*NU-1)-Z(2*NU+1))**2 )
C
       IF(ABS(G(NU)).GT.1.E-12)GO TO 41
C
C           G .LT. 1.E-12 IMPLIES NO RELATIVE MOTION
C           BETWEEN ADJACENT MASSES (PLATFORMS)
C
       IF (ABS(G(NU)) .LT. 1.E-12) INDG(NU)=1
C
C        ...IF G=0 AND ANY RELATED Z=0
C              THEN NEITHER MASS HAS MOTION...
C
```

```
      IF (     (INDG(NU)          .EQ.  1)          .AND.
     1         (ABS(Z(2*NU))      .LT. 1.E-14)      .AND.
     2         (ABS(Z(2*NU-1))    .LT. 1.E-14)   )    INDG(NU)=-1
C
      IF (INDG(NU) .EQ.  1) IND(NU)=1
      IF (INDG(NU) .EQ.  1) IND(NU+1)=1
C
      W(NU) = 0.
      GO TO 42
C
   41 CONTINUE
      W(NU) = (4.  * MU * N(NU)) / (PI*G(NU))
C
   42 CONTINUE
      IF (ABA(NU) .GT.  1.E-12) GO TO 43
      V(NU) = 0.
      GO TO 44
C
   43 CONTINUE
      V(NU) = (4.  * NN(NU) * MU) / (PI * ABA(NU))
C
   44 CONTINUE
   45 CONTINUE
C
     IF (ITNO .LT.  ITMIN) GO TO 52
     IF (ITNO .EQ.       1) GO TO 52
C
     DIFF1 = 0.
     DIFF  = 0.
C
     DO 50  NU=1,ML
      Z1MZ = ABS(Z1(NU)-Z(NU))
      IF (Z1MZ .GT.  DIFF) DIFF=Z1MZ
      FAC7 = DIFF1
      ZED = 0.
      IF (Z1(NU) .NE.  ZED) FAC7=Z1MZ/ABS(Z1(NU))
      IF (FAC7 .GT.  DIFF1) DIFF1=FAC7
   50 CONTINUE
C
     IF (DIFF  .LT.   EPS) TEST=.TRUE.
     IF (DIFF1 .LT.  EPS1) TEST1=.TRUE.
     IF (TEST .OR.  TEST1) GO TO 56
     IF (ITNO .GE.  ITMAX) GO TO 56
C
   52 CONTINUE
C
     DO 55  NU=1,ML
   55 Z1(NU) = Z(NU)
     IF (ITNO .GT.  1) GO TO 11
     IF (.NOT.  KEY   ) GO TO 11
C
   56 CONTINUE
C
C......   CHECK FOR OUTPUT HEADINGS..
C
     IF (IPERS .NE.  0) THEN
       FACTA = ((ISTEP-1)/IPERS)*IPERS + 1
       IF (FACTA .EQ.  ISTEP) CALL PAGER
       CALL OMHEAD
     END IF
C
```

```
      JLINE = 0
C
      DO 70  I=1,NL
C
       WONE=1.0
       A(I)=SQRT(Z(2*I-1)**2+Z(2*I)**2)
       X=Z(2*I-1)
       Y=-Z(2*I)
       IF(ABS(X).LT.0.00001*Y)ALPHA(I)=SIGN(WONE,Y)*PI/2.
       IF(ABS(X).GT.0.00001*Y)ALPHA(I)=ATAN2(Y,X)
C
C       ...RECOVER 'D' AND 'GAMMA'...
C
      D(I) = 1./ SQRT( (K(1,I)-M1(I)*OMEGA**2)**2
     1                 + (K(1,I) * ETA1)**2 )
C
      SUM = S(I)**2
C
      IF (IND(I) .EQ. 0) THEN
        SUM = SUM + A(I)**2 * K(1,I)**2 * (1.+ETA1**2) +
     1        2. * A(I) * S(I) * K(1,I) *
     2        (COS(DELTA(I) - ALPHA(I)) +
     3              ETA1 * SIN(DELTA(I) - ALPHA(I)))
      END IF
C
      D(I) = D(I) * SQRT(SUM)
C
      SUM1 = S(I) * ((K(1,I)-M1(I)*OMEGA**2) * SIN(DELTA(I))
     1              - K(1,I) * ETA1 * COS(DELTA(I)))
C
      IF (IND(I) .EQ. 0) THEN
        SUM1 = SUM1 + A(I) * K(1,I) * SIN(ALPHA(I)) *
     1         (K(1,I) * (1. + ETA1**2) - M1(I) * OMEGA**2)
     2         - A(I) * COS(ALPHA(I)) * M1(I) * K(1,I) *
     3         ETA1 * OMEGA**2
      END IF
C
      SUM2 = S(I) * ((K(1,I) - M1(I) * OMEGA**2) * COS(DELTA(I))
     1        + K(1,I) * ETA1 * SIN(DELTA(I)))
C
      IF (IND(I) .EQ. 0) THEN
        SUM2 = SUM2 + A(I) * K(1,I) * COS(ALPHA(I)) *
     2         (K(1,I) * (1. + ETA1**2) - M1(I) * OMEGA**2) +
     3         A(I) * SIN(ALPHA(I)) * M1(I) * K(1,I) * ETA1 * OMEGA**2
      END IF
C
      GAMMA(I) = ATAN2(SUM1,SUM2)
C
  703 FORMAT (I2, 4E20.10)
C
C
C.....   CHECK FOR OUTPUT FORMATTING  ...
C
      IF (IPERS .EQ. 0) THEN
        JLINE = JLINE + 1
        IF ( (I .EQ. 1)  .OR.
     1       (JLINE .GT. JPERS) ) THEN
          JLINE = 1
          CALL PAGER
          CALL OMHEAD
        END IF
```

```
      END IF
C
C   ...   END CHECK.
C
      ALPOUT(I) = (ALPHA(I)*180.)/PI
      GAMOUT(I) = (GAMMA(I)*180.)/PI
C
      AACC(I)   = A(I)*GFAC1
      DACC(I)   = D(I)*GFAC1
C
      WRITE (6,610) I, A(I), ALPOUT(I), D(I), GAMOUT(I),
     1               Z(2*I-1), Z1(2*I-1)
C
  610 FORMAT (3X, I5, 2X, 2(4X, 1PE15.8, 4X, 0PF11.3, 4X),
     1        3X, 2(1X, 1PE19.12) )
C
      WRITE (6,613) AACC(I), DACC(I), Z(2*I), Z1(2*I)
C
C  613 FORMAT (  89X, 2(1X, 1PE19.12) )
  613 FORMAT (14X,1PE15.8,23X,1PE15.8,22X,2(1X, 1PE19.12))
C
      IF (INDG(I) .EQ. -1) WRITE (6,612) I, I+1
  612 FORMAT (' ', 85X, 'SOLID MOTION FOR ', I2, ' AND ', I2 )
C
   70 CONTINUE
C
C-------------WRITE BPOST VALS------------
C
      IF (IBPOST .NE. 1) GO TO 516
  515 WRITE (51, 6515) ISTEP, OMOUT, (A(II)      ,II=1,NL)
      WRITE (52, 6515) ISTEP, OMOUT, (D(II)      ,II=1,NL)
      WRITE (53, 6515) ISTEP, OMOUT, (ALPOUT(II),II=1,NL)
      WRITE (54, 6515) ISTEP, OMOUT, (GAMOUT(II),II=1,NL)
      WRITE (55, 6515) ISTEP, OMOUT, (AACC(II)   ,II=1,NL)
      WRITE (56, 6515) ISTEP, OMOUT, (DACC(II)   ,II=1,NL)
 6515 FORMAT (I10, 51(1PE14.5) )
  516 CONTINUE
C
C---------------------------------------
C
      IF (ITNO .EQ. 1) GO TO 11
      IF (OMEGA .LT. OMGEND) GO TO 1
  100 CONTINUE
C
  518 CALL PAGER
      IF (IBPOST .NE. 1) GO TO 519
      CLOSE (51)
      CLOSE (52)
      CLOSE (53)
      CLOSE (54)
      CLOSE (55)
      CLOSE (56)
  519 CONTINUE
C
C-------------------------
      STOP
      END
      SUBROUTINE INALL (NBL, X)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      common /pars/ iread,ikm,isd
```

```
C
      DIMENSION X(NBL)
C
      CHARACTER*20 STR
C
  100 READ (9, 5100) STR
      IREAD = NBL
C
      DO 110  I=1,18
        IP2 = I+2
  110   IF (STR(I:IP2) .EQ. 'ALL') IREAD=1
C
  120 READ (9, *) (X(I), I=1,IREAD)
C
      IF (IREAD .EQ. 1) THEN
        DO 130  I=1,NBL
  130   X(I) = X(1)
      END IF
C
      RETURN
C
 5100 FORMAT (1X, A20)
C
      END
C
      SUBROUTINE INDATA
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      CHARACTER*82 HEADER
      CHARACTER*50 TITLE, BLNK50
      CHARACTER*25 BDP
      CHARACTER*6 AC
      CHARACTER*1  BLNK1
C
      COMMON /BLK1/ NL, M1, K, M2, N, S, DELTA, NN, OMEGAO,
     1              DOMEGA, OMGEND, ETA1, ETA3, AL, MU, AOMEGA,
     2              GRAVIT, DFACT, IDELTA, ETA2
C
      COMMON /BLK2/ HEADER, TITLE, BLNK50, BDP, BLNK1, AC
      COMMON /BLK3/ IPAGE, ISTEP, OMOUT, ITNO, DIFF, DIFF1
C
      COMMON /BLK4/ ITMIN, ITMAX, EPS, EPS1, TLIM, ZLIM
      COMMON /PARS/ IREAD,IKM,ISD
C
      DIMENSION AL(50), K(3,50), M1(50), M2(50), N(50), NN(50)
      DIMENSION DELTA(50), S(50), DFACT(50)
C
      DIMENSION KDUM(50)
C
      REAL*8   M1, M2, K, NN, N, MU
      REAL*8   KDUM
C
      DATA PI/3.1415926535898/
C
  100 READ (9, 5100) TITLE
      CALL HEDSET
      IPAGE = 0
      CALL PAGER
  105 WRITE (6, 6105)
  110 WRITE (6, 6110) TITLE
```

```
C
  120 READ (9, *) NL
  130 WRITE (6, 6130) NL
      IF ( (NL .LT.   1) .OR.
     +      (NL .GT.  50)) THEN
  135   WRITE (6, 6135)
        STOP
      END IF
C
  140 READ (9, *) OMEGAO, OMGEND, DOMEGA
  150 WRITE (6, 6150) OMEGAO, OMGEND, DOMEGA
C
C......   CONVERT TO RADIANS...
C
      AOMEGA = OMEGAO
      OMEGAO = OMEGAO * 2.  * PI
      OMGEND = OMGEND * 2.  * PI
      DOMEGA = DOMEGA * 2.  * PI
C
C   ...   CONVERSION COMPLETE.
C
C
  160 READ (9, *) ITMIN, ITMAX, EPS, EPS1, ZLIM
  170 WRITE (6, 6170) ITMIN, ITMAX, EPS, EPS1, ZLIM
C
  180 READ (9, *) TLIM
  190 WRITE (6, 6190) TLIM
C
  200 READ (9, *) ETA1, ETA2, ETA3, MU
  210 WRITE (6, 6210) ETA1, ETA2, ETA3, MU
C
```

```
  212 READ (9, *) GRAVIT
  214 WRITE (6, 6214) GRAVIT
C
C---SET DELTA CONSTANT WITH FREQ.  TH 2/25/84
C
      IDELTA = 0
C
C---  216 READ (9, *) IDELTA
C---         IF (IDELTA .NE. 1) THEN
C---           IDELTA = 0
C---  217      WRITE (6, 6217)
C---         ELSE
C---  218      WRITE (6, 6218)
C---         END IF
C
      CALL PAGER
  220 WRITE (6, 6220)
C
      WRITE (6, 6500)
      CALL INALL (NL, M1)
      IKM=IREAD
      AC = 'M1    ='
      CALL OUTALL (NL,M1)
C
      WRITE (6, 6500)
      CALL INALL (NL, M2)
      IKM=IKM+IREAD
      AC = 'M2    ='
      CALL OUTALL (NL,M2)
C
      WRITE (6, 6500)
      CALL INALL (NL,KDUM)
      IKM=IKM+IREAD
      AC = 'K1    ='
      CALL OUTALL (NL,KDUM)
      DO 250  I=1,NL
  250 K(1,I) = KDUM(I)
C
      WRITE (6, 6500)
      CALL INALL (NL, KDUM)
      IKM=IKM+IREAD
      AC = 'K2    ='
      CALL OUTALL (NL,KDUM)
      DO 260  I=1,NL
  260 K(2,I) = KDUM(I)
C
      WRITE (6, 6500)
      CALL INALL (NL, KDUM)
      IKM=IKM+IREAD
      AC = 'K3    ='
      CALL OUTALL (NL,KDUM)
      DO 270  I=1,NL
  270 K(3,I) = KDUM(I)
C
      WRITE (6, 6500)
      CALL INALL (NL, S)
      ISD=IREAD
      AC = 'S     ='
      CALL OUTALL (NL,S)
C
      IF (IDELTA .EQ. 0) THEN
```

```
          WRITE (6, 6500)
          CALL INALL (NL, DELTA)
          ISD=ISD+IREAD
          AC = 'DELTA='
          CALL OUTALL (NL,DELTA)
        ELSE
          WRITE (6, 6500)
          CALL INALL (NL, DFACT)
          ISD=ISD+IREAD
          AC = 'DFACT='
          CALL OUTALL (NL, DFACT)
        END IF
C
        WRITE (6, 6500)
        CALL INALL (NL, NN)
        AC = 'NN   ='
        CALL OUTALL (NL,NN)
C
        WRITE (6, 6500)
        CALL INALL (NL, N)
        AC = 'N    ='
        CALL OUTALL (NL,N)
C
        RETURN
C
 5100 FORMAT (1X,A50)
C
 6105 FORMAT ( //5X,  'INPUT PARAMETERS',
     1              /5X,  '----------------'    )
C
 6110 FORMAT ( //10X,  'PROBLEM DESCRIPTION :  ', A50)
C
 6130 FORMAT ( //10X,  'NUMBER OF BLADES.........:', I4 )
C
 6135 FORMAT ( //5X,
     +    //5X 'FATAL ERROR IN INPUT DATA.',
     +     /5X,  'MINIMUM ALLOWED NUMBER OF BLADES :   1',
     +     /5X,  'MAXIMUM ALLOWED NUMBER OF BLADES :  50',
     +    //5X,  'THE PROGRAM SOURCE CODE MUST BE CHANGED',
     +     /5X,  'TO ANALYZE PROBLEMS WITH MORE THAN THE',
     +     /5X,  'MAXIMUM NUMBER OF BLADES.  ', ////         )
C
 6150 FORMAT ( //10X,  'INPUT FREQUENCIES :',
     1            //10X,  '  1.  INITIAL OMEGA.......:', 1PE12.5,
     2            //10X,  '  2.  FINAL OMEGA.........:', 1PE12.5,
     3            //10X,  '  3.  DELTA OMEGA.........:', 1PE12.5    )
C
 6170 FORMAT ( //10X,  'ITERATION PARAMETERS :',
     1            //10X,  '  1.  MINIMUM ITERATIONS :', I4,
     2            //10X,  '  2.  MAXIMUM ITERATIONS :', I4,
     3            //10X,  '  3.  EPS.................:', 1PE12.5,
     4            //10X,  '  4.  EPS1................:', 1PE12.5   ,
     5            //10X,  '  5.  ZLIM................:', 1PE12.5   )
C
 6190 FORMAT ( //10X,  'PROGRAM RUN-TIME LIMIT  :', F10.1   )
C
 6210 FORMAT ( //10X,  'MODEL PARAMETERS :',
     1            //10X,  '  1.  ETA1................:', 1PE12.5,
     2            //10X,  '  2.  ETA2................:', 1PE12.5,
     3            //10X,  '  3.  ETA3................:', 1PE12.5    ,
     4            //10X,  '  4.  MU..................:', 1PE12.5    )
```

```
C
 6214 FORMAT ( //10X, 'GRAVITATIONAL CONSTANT  :', 1PE12.5    )
C
 6217 FORMAT ( //10X, 'DELTA : DOES NOT VARY WITH FREQUENCY' )
C
 6218 FORMAT ( //10X, 'DELTA : VARIES WITH FREQUENCY'        )
C
 6220 FORMAT ( //5X, 'INPUT PARAMETERS (CONT''D)',
     1             /5X, '-------------------------'   )
C
 6500 FORMAT ( // )
C
      END
C
      SUBROUTINE HEDSET
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      CHARACTER*82 HEADER
      CHARACTER*50 TITLE, BLNK50
      CHARACTER*25 BDP
      CHARACTER*6 AC
      CHARACTER*1  BLNK1
C
      COMMON /BLK2/ HEADER, TITLE, BLNK50, BDP, BLNK1, AC
C
C......    SET INITIAL VALUES.
C
      BLNK1 = ' '
      BLNK50 = ' '
      HEADER = ' '
C
      LBDP = 25
      BDP = 'BLADE DYNAMICS PROGRAM - '
C
C......    FIND NON-BLANK TITLE SUBSTRING.
C
      IF (TITLE .EQ. BLNK50) THEN
        TITLE = 'NO TITLE SPECIFIED'
      END IF
C
      IFIRST = 0
      ILAST  = 0
C
      DO 50  I=1,50
C
        IF ( (IFIRST .EQ. 0)  .AND.
     1        (TITLE(I:I) .NE. BLNK1) ) IFIRST=I
C
        IBACK = 51-I
        IF ( (ILAST  .EQ. 0)  .AND.
     1        (TITLE(IBACK:IBACK) .NE. BLNK1) ) ILAST=IBACK
C
   50 CONTINUE
C
      LENGTH = ILAST-IFIRST+1
C
C......    CENTER HEADER STRINGS.
C
      ITOT = LBDP + LENGTH
C
```

```
      JA = ( (82-ITOT)/2 ) + 1
      JAPLUS = JA + LBDP - 1
C
      JB = JAPLUS + 1
      JBPLUS = JB + LENGTH - 1
C
      HEADER(JA:JAPLUS) = BDP
      HEADER(JB:JBPLUS) = TITLE(IFIRST:ILAST)
C
      RETURN
      END
C
      SUBROUTINE LOAD (X,Y,N,S,M,T)
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      DIMENSION X(N), Y(N)
C
      IF (T .NE. 0. ) GO TO 15
C
      DO 10  I=1,N
   10 Y(I) = X(I) * 10.**(S*M)
      RETURN
C
   15 DO 20  I=1,N
   20 Y(I) = T
      RETURN
C
      END
C
      SUBROUTINE MATOUT (VEC,PLR,NUM,ITNO)
C
      CHARACTER*3 PLR
      DIMENSION VEC(*)
C
  100 WRITE (6,6100) PLR, ITNO
 6100 FORMAT (//5X, A3, ' VECTOR AT ITNO = ', I5, /)
C
      IFF = -4
  110 IFF = IFF + 5
      ILL = MIN (NUM, (IFF+4) )
  120 WRITE (6,6120) (PLR,JFL,VEC(JFL),JFL=IFF,ILL)
 6120 FORMAT (2X, A3, 5(I3, '=', 1PE14.6, A3) )
C
      IF (ILL .LT. NUM) GO TO 110
      RETURN
      END
C
      SUBROUTINE OMHEAD
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      CHARACTER*82 HEADER
      CHARACTER*50 TITLE,BLNK50
      CHARACTER*25 BDP
      CHARACTER*6 AC
      CHARACTER*1  BLNK1
C
      COMMON /BLK3/ IPAGE, ISTEP, OMOUT, ITNO, DIFF, DIFF1
C
```

```
      IF (ITNO .EQ. 1) GO TO 100
C
   10 WRITE (6,610) ISTEP, OMOUT, ITNO, DIFF, DIFF1
C
  610 FORMAT ( //4X, 'STEP=', I4, 4X, 'OMEGA=', F15.5,
     1        10X, 'ITERATIONS=', I4, 4X, 'ABS. Z DIFF=',
     2        1PE15.8, 4X, 'REL. Z DIFF=', 1PE15.8, /)
C
      WRITE (6, 650)
C
      RETURN
C
  100 WRITE (6,620) ISTEP, OMOUT
C
  620 FORMAT ( //4X, 'STEP=', I4, 'OMEGA=', 1PE15.8,
     1          '     --- NO FRICTION --', /)
C
      WRITE (6, 650)
C
  650 FORMAT (   5X, 'BLADE', 4X, 7X, 'A', 7X,
     1           4X, 5X, 'ALPHA', 5X,
     2           4X, 7X, 'D', 7X, 4X, 5X, 'GAMMA', 5X,
     3           4X, 2X, 'Z(2*I-1)//Z(2*I)', 1X,
     4           1X, 1X, 'Z1(2*I-1)//Z1(2*I)' ,
     5     /5X, 5('-'), 4(4X, 15('-')), 3X, 2(1X, 19('-')) )
C
      RETURN
      END
C
      SUBROUTINE OUTALL (NBL,X)
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      CHARACTER*82 HEADER
      CHARACTER*50 TITLE, BLNK50
      CHARACTER*25 BDP
      CHARACTER*6  AC
      CHARACTER*1  BLNK1
C
      COMMON /BLK2/ HEADER, TITLE, BLNK50, BDP, BLNK1, AC
C
      DIMENSION X(*)
C
      NCOLS = 10
C
      IFULL = NBL/NCOLS
      IREM = NBL - (IFULL*NCOLS)
C
      INDP = 0
      IF (IFULL .EQ. 0) GO TO 50
        DO 20  I=1,IFULL
          IND = INDP + 1
          INDP = IND + NCOLS - 1
   10     WRITE (6, 610) AC, (X(J), J=IND,INDP)
   20   CONTINUE
C
   50 IF (IREM .EQ. 0) GO TO 100
        IND = INDP + 1
        INDP = INDP + IREM
   60   WRITE (6, 610) AC, (X(J), J=IND,INDP)
C
```

```
  100 RETURN
C
  610 FORMAT (4X, A6, 10(1PE12.5) )
C
      END
C
      SUBROUTINE PAGER
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      CHARACTER*82 HEADER
C
      COMMON /BLK2/ HEADER
      COMMON /BLK3/ IPAGE
C
      IPAGE = IPAGE + 1
C
   10 WRITE (6, 610) IPAGE, HEADER, IPAGE
C
  610 FORMAT (  '1', //, '   ***** PAGE', I5, ' *****   ',
     1           A82, '  ***** PAGE', I5, ' *****', /)
C
      RETURN
      END
C
      SUBROUTINE QFIX(Q,ML)
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      DIMENSION Q(*)
      COMMON /BLK4/ ITMIN, ITMAX, EPS, EPS1, TLIM, ZLIM
C
      DO 100  I=1,ML
      ABSQ = ABS(Q(I))
      IF (ABSQ .LE. ZLIM) Q(I) = 0.
  100 CONTINUE
C
      RETURN
      END
```

```
IMSL ROUTINE NAME   - LEQ2S

PURPOSE             - LINEAR EQUATION SOLUTION - INDEFINITE MATRIX
                        - SYMMETRIC STORAGE MODE - HIGH ACCURACY
                        SOLUTION

USAGE               - CALL LEQ2S (A,N,B,M,IB,IJOB,ICHNG,DET,IER)

ARGUMENTS    A      - THE COEFFICIENT MATRIX OF THE EQUATION
                        AX = B, WHERE A IS ASSUMED TO BE AN N BY N
                        SYMMETRIC MATRIX. A IS STORED IN SYMMETRIC
                        STORAGE MODE AND THEREFORE HAS DIMENSION
                        N*(N+1)/2. (INPUT)
             N      - ORDER OF A AND THE NUMBER OF ROWS IN B.
                        (INPUT)
             B      - INPUT/OUTPUT MATRIX OF DIMENSION N BY M.
                      ON INPUT, B CONTAINS THE M RIGHT HAND SIDES
                        OF THE EQUATION AX = B.
                      ON OUTPUT, THE SOLUTION MATRIX X REPLACES B.
                        IF IJOB = 1, B IS NOT USED.
             M      - NUMBER OF RIGHT HAND SIDES (COLUMNS IN B).
                        (INPUT)
             IB     - ROW DIMENSION OF MATRIX B EXACTLY AS
                        SPECIFIED IN THE DIMENSION STATEMENT IN THE
                        CALLING PROGRAM. (INPUT)
             IJOB   - INPUT OPTION PARAMETER. IJOB = I IMPLIES:
                        I = 0, FACTOR THE MATRIX A AND SOLVE THE
                          EQUATION AX = B.
                        I = 1, FACTOR THE MATRIX A. THE FACTORIZED
                          FORM OF A IS STORED IN THE FIRST
                          N*(N+1)/2 LOCATIONS OF DET.
                        I = 2, SOLVE THE EQUATION AX = B. THIS
                          OPTION IMPLIES THAT MATRIX A HAS ALREADY
                          BEEN FACTORED BY LEQ2S USING
                          IJOB = 0 OR 1. IN THIS CASE, THE
                          INFORMATION CONTAINED IN DET AND ICHNG
                          MUST HAVE BEEN SAVED FOR REUSE IN THE
                          CALL TO LEQ2S.
             ICHNG  - WORK AREA OF LENGTH 2N.
             DET    - WORK AREA OF LENGTH N*(N+1)/2+3N.
             IER    - ERROR PARAMETER. (OUTPUT)
                      TERMINAL ERROR
                        IER = 129 INDICATES THAT MATRIX A IS
                          ALGORITHMICALLY SINGULAR. (SEE THE
                          CHAPTER L PRELUDE)
                        IER = 130 INDICATES THAT ITERATIVE
                          IMPROVEMENT FAILED TO CONVERGE. THE
                          MATRIX IS TOO ILL-CONDITIONED.

PRECISION/HARDWARE  - SINGLE AND DOUBLE/H32
                    - SINGLE/H36,H48,H60

REQD. IMSL ROUTINES - SINGLE/LEQ1S,UERTST,UGETIO
                    - DOUBLE/LEQ1S,UERTST,UGETIO,VXADD,VXMUL,
                        VXSTO
```

RESPONSE PLOT PROGRAM BPLOT.FOR

```
C
      PROGRAM BPLOT
C
      COMMON /INP1/ OMINIT(20), OMLAST(20),
     1              ICPS, ITERM
      COMMON /INP2/ IRUNTL(20,15), ITITLE(50), IDUMMY(15)
C
      COMMON /INP3/ MAXPTS(20), MAXBLD(20), NBL(20)
      COMMON /INP4/ NFILES, NCURVE(20), ITOTCV, NBLADE(20,50)
      COMMON /INP5/ FNAME(20)
C
      COMMON /LABS/ IXLAB(30), IYLAB(30)
      COMMON /CHAR/ ICHAXS, ICHTIT, JXSPAC, KYSPAC, LTSPAC
C
      COMMON /AXS / XAXMIN, XAXMAX, YAXMIN, YAXMAX
      COMMON /TRAN/ IXTRAN, IYTRAN
C
      COMMON /SCRN/ IXSCMN, IXSCMX, IYSCMN, IYSCMX,
     1              IMINTX, IMINTY, IMAJTX, IMAJTY
C
      COMMON /LEGS/ LFLAG,IDESC(1000),JDESC(1000,20),MXDESC,JCV
      CHARACTER * 100 FNAME
C
C
C...DO PLOTS.
C
      CALL FSET
      CALL SETTEK
  100 DO 110 J=1,NFILES
         CALL OUTSUM (J,0)
  110 CONTINUE
      CALL INPLOT
  120 CALL PLODER
      CALL TRIM
      CALL LEGDRW
      IF (ITERM .EQ. 3) CALL CHRSIZ(4)
C
C...CHECK FOR REDRAW (USER BLEW LEGEND)
C
      CALL DRCHEK (IANS)
      IF (IANS .EQ. 1HY) GO TO 120
      CALL FNPLOT
C
C...CHECK FOR NEW PLOT, SAME FILES.
C
  200 WRITE (6,6200)
      READ (5,5200,ERR=200) IANS
      IF (IANS .EQ. 1HY) GO TO 100
      STOP
C
 6200 FORMAT (/2X, 'SAME FILES, NEW PLOT? (Y/N) ...: ' $)
C
 5200 FORMAT (A1)
C
      END
C
      SUBROUTINE FSET
C
      COMMON /INP1/ OMINIT(20), OMLAST(20),
     1              ICPS, ITERM
      COMMON /INP2/ IRUNTL(20,15), ITITLE(50), IDUMMY(15)
C
```

```
      COMMON /INP3/ MAXPTS(20), MAXBLD(20), NBL(20)
      COMMON /INP4/ NFILES, NCURVE(20), ITOTCV, NBLADE(20,50)
      COMMON /INP5/ FNAME(20)
C
      COMMON /LABS/ IXLAB(30), IYLAB(30)
      COMMON /CHAR/ ICHAXS, ICHTIT, JXSPAC, KYSPAC, LTSPAC
C
      COMMON /AXS / XAXMIN, XAXMAX, YAXMIN, YAXMAX
      COMMON /TRAN/ IXTRAN, IYTRAN
C
      COMMON /SCRN/ IXSCMN, IXSCMX, IYSCMN, IYSCMX,
     1              IMINTX, IMINTY, IMAJTX, IMAJTY
C
      COMMON /LEGS/ LFLAG,IDESC(1000),JDESC(1000,20),MXDESC,JCV(1
      CHARACTER * 100 FNAME
C
      MAXFIL = 20
  100 WRITE (6,6100) MAXFIL
      READ (5,*,ERR=100) NFILES
C
      IF (NFILES .GT. MAXFIL) NFILES=MAXFIL
C
C
 6100 FORMAT (/2X, 'MAX INPUT FILES.................:  ', I2,
     1        /2X,'HOW MANY INPUT FILES?..........:  ' $)
      DO 300 I=1,NFILES
         IUNIT = 50 + I
  250    WRITE (6,6000) I
         READ  (5,7000,ERR=250) FNAME(I)
         OPEN  (UNIT=IUNIT,FILE=FNAME(I),ACCESS='SEQUENTIAL',
     +          FORM='FORMATTED',
     +          STATUS='OLD',ERR=260)
          GO TO 300
  260     WRITE (6, 6260) FNAME(I)
 6260 FORMAT (//2X,  '* * ERROR IN OPENING FILE :  ', A, /)
          GO TO 250
  300    CONTINUE
      RETURN
 6000 FORMAT (/2X,'ENTER FILENAME NO. ',I3,' .........:  ',$)
 7000 FORMAT ( A )
      END
C
      SUBROUTINE SETLEG
C
      COMMON /INP1/ OMINIT(20), OMLAST(20),
     1              ICPS, ITERM
      COMMON /INP2/ IRUNTL(20,15), ITITLE(50), IDUMMY(15)
C
      COMMON /INP3/ MAXPTS(20), MAXBLD(20), NBL(20)
      COMMON /INP4/ NFILES, NCURVE(20), ITOTCV, NBLADE(20,50)
      COMMON /INP5/ FNAME(20)
C
      COMMON /LABS/ IXLAB(30), IYLAB(30)
      COMMON /CHAR/ ICHAXS, ICHTIT, JXSPAC, KYSPAC, LTSPAC
C
      COMMON /AXS / XAXMIN, XAXMAX, YAXMIN, YAXMAX
      COMMON /TRAN/ IXTRAN, IYTRAN
C
      COMMON /SCRN/ IXSCMN, IXSCMX, IYSCMN, IYSCMX,
     1              IMINTX, IMINTY, IMAJTX, IMAJTY
C
```

```
      COMMON /LEGS/ LFLAG, IDESC(1000), JDESC(1000, 20), MXDESC, JCV(10)
      CHARACTER * 100 FNAME
      DIMENSION JDUMMY(20)
C
      LFLAG = 0
      MXDESC = 0
   10 WRITE (6, 6010)
      READ (5, 5010, ERR=10) IANS
C
      IF (IANS .NE. 1HY) RETURN
C
      LFLAG = 1
C
      IMAJTX = 4
      IMAJTY = 4
C
      DO 50  II=1, ITOTCV
   30   WRITE (6, 6030) II
        READ (5, 5030, ERR=30) (JDUMMY(JJ), JJ=1, 20)
C
        CALL NCHARX (JDUMMY, KK, 20)
C
        IF (MXDESC .LT. KK) MXDESC=KK
        IDESC(II) = KK
C
        DO 40  LL=1, 20
   40   JDESC(II, LL) = JDUMMY(LL)
C
   50 CONTINUE
C
   60 CONTINUE
C
   70 WRITE (6, 6070)
C
      RETURN
C
 5010 FORMAT (A1)
 5030 FORMAT (20A1)
C
 6010 FORMAT (/2X, 'DRAW A LEGEND? (Y/N)............: ' $)
C
 6030 FORMAT (/2X, 'CURVE (', I2, ') DESCRIPTION.....: ' $)
C
 6070 FORMAT (/2X, 'WHEN PLOT IS DONE, SET CURSOR',
     1       /2X, 'FOR UPPER LEFT CORNER OF LEGEND',
     2       /2X, 'BOX AND TYPE A SINGLE CHARACTER. ', //)
C
      END
C
      SUBROUTINE MINMAX (YY, NB, AMNX, IAMNX)
C
      DIMENSION AMNX(2), IAMNX(2), YY(12)
C
      DO 100  I=1, NB
        IF (YY(I) .LT. AMNX(1)) THEN
          IAMNX(1) = I
          AMNX(1)  = YY(I)
        END IF
        IF (YY(I) .GT. AMNX(2)) THEN
          IAMNX(2) = I
          AMNX(2)  = YY(I)
```

```
         END IF
   100 CONTINUE
       RETURN
       END
C
       SUBROUTINE SETJCV
C
       COMMON /INP1/ OMINIT(20), OMLAST(20),
      1              ICPS, ITERM
       COMMON /INP2/ IRUNTL(20,15), ITITLE(50), IDUMMY(15)
C
       COMMON /INP3/ MAXPTS(20), MAXBLD(20), NBL(20)
       COMMON /INP4/ NFILES, NCURVE(20), ITOTCV, NBLADE(20,50)
       COMMON /INP5/ FNAME(20)
C
       COMMON /LABS/ IXLAB(30), IYLAB(30)
       COMMON /CHAR/ ICHAXS, ICHTIT, JXSPAC, KYSPAC, LTSPAC
C
       COMMON /AXS / XAXMIN, XAXMAX, YAXMIN, YAXMAX
       COMMON /TRAN/ IXTRAN, IYTRAN
C
       COMMON /SCRN/ IXSCMN, IXSCMX, IYSCMN, IYSCMX,
      1              IMINTX, IMINTY, IMAJTX, IMAJTY
C
       COMMON /LEGS/ LFLAG,IDESC(1000),JDESC(1000,20),MXDESC,JCV(10)
       CHARACTER * 100 FNAME
C
C----THIS ROUTINE SETS VALUES INTO JCV FOR USE----
C     IN THE PLOT 10 'DASHA' CALL, LINE TYPE
C     ARGUMENT.
C
       JCV ( 1) = 0
       JCV ( 2) = 36
       JCV ( 3) = 3676
       JCV ( 4) = 56
       JCV ( 5) = 76
       IF (ITERM .NE. 3) GO TO 100
C
         JCV ( 1) = 0
         JCV ( 2) = 1
         JCV ( 3) = 2
         JCV ( 4) = 3
         JCV ( 5) = 4
C
   100 CONTINUE
C
       JCV ( 6) = 3656
       JCV ( 7) = 367676
       JCV ( 8) = 5676
       JCV ( 9) = 363656
       JCV (10) = 363676
C
       RETURN
       END
C
       SUBROUTINE OUTSUM (J,IND)
C
C    OUTSUM CHECKS THE FILE JF (JF=J+50) FOR MINS AND
C   MAXS OVER AN INTERVAL CORRESPONDING TO INPUT IND
C    IF IND = 0 : CHECK ALL X-VALUES IN FILE JF
C           = 1 : CHECK OVER RANGE OF COMMON XAXMIN-MAX
```

```
C
C---
      COMMON /INP1/ OMINIT(20), OMLAST(20),
     1              ICPS, ITERM
      COMMON /INP2/ IRUNTL(20,15), ITITLE(50), IDUMMY(15)
C
      COMMON /INP3/ MAXPTS(20), MAXBLD(20), NBL(20)
      COMMON /INP4/ NFILES, NCURVE(20), ITOTCV, NBLADE(20,50)
      COMMON /INP5/ FNAME(20)
C
      COMMON /LABS/ IXLAB(30), IYLAB(30)
      COMMON /CHAR/ ICHAXS, ICHTIT, JXSPAC, KYSPAC, LTSPAC
C
      COMMON /AXS / XAXMIN, XAXMAX, YAXMIN, YAXMAX
      COMMON /TRAN/ IXTRAN, IYTRAN
C
      COMMON /SCRN/ IXSCMN, IXSCMX, IYSCMN, IYSCMX,
     1              IMINTX, IMINTY, IMAJTX, IMAJTY
C
      COMMON /LEGS/ LFLAG,IDESC(1000),JDESC(1000,20),MXDESC,JCV(10)
      CHARACTER * 100 FNAME
C
C---
      DIMENSION AA(50), AAOLD(50)
      DIMENSION AMNMX(2), IAMNMX(2), JPOINT(2)
C
C-----MINS AND MAXES : 1--MIN ; 2--MAX
C
      JF = J + 50
      REWIND JF
C
      IF (IND .EQ. 0) GO TO 55
   30   WRITE (6,6030)
   35   READ (5,5035,ERR=30) IANS
        IF (IANS .EQ. 1HN) RETURN
C--   50   WRITE (6,6050) JF
        JFM50 = JF-50
   50   WRITE (6, 6050) JFM50, FNAME(JFM50)
   55 CONTINUE
C
      AMNMX(1) =  1.E20
      AMNMX(2) = -1.E20
      IAMNMX(1)= 0
      IAMNMX(2)= 0
      JPOINT(1)= 0
      JPOINT(2)= 0
C
      IPOINT = 0
C
  100 READ (JF,5100) (IDUMMY(II),II=1,15)
 5100 FORMAT (15A4)
C
  110 READ (JF,5110) NBL(J)
 5110 FORMAT (I10)
      NB = NBL(J)
C
C-----READ FIRST X-VALS, Y-VALS, 5 BRANCHES----
C
  200 READ (JF,5200,END=600) JDUM, XNEW, (AA(II),II=1,NB)
 5200 FORMAT (I10, 51(1PE14.5) )
C
```

```
      IPOINT = IPOINT + 1
C
        IF (JDUM .EQ. -999) GO TO 600
        IF (IND  .EQ.    0) GO TO 220
        IF (XNEW .GT. XAXMIN) GO TO 230
        IF (XNEW .EQ. XAXMIN) GO TO 240
C
C----ELSE, KEEP LOOKING----
C
        JPOINT(1) = JDUM
        JPOINT(2) = JDUM
        XOLD = XNEW
        DO 210  II=1,NB
          AAOLD(II) = AA(II)
  210   CONTINUE
        GO TO 200
C
C----FIRST X IN RANGE FOUND----
C
  220 XAXMIN = XNEW
      XAXMAX = XNEW
      OMINIT(J) = XAXMIN
      DO 225  II=1,NB
        AAOLD(II) = AA(II)
  225 CONTINUE
      GO TO 240
C
  230 IF (XNEW .LT. XAXMAX) GO TO 232
      IF (JPOINT(1) .EQ. 0) GO TO 650
        JPOINT(2) = JDUM
        CALL MINMAX (AA,NB,AMNMX,IAMNX)
        GO TO 500
C
  232 CALL MINMAX (AAOLD,NB,AMNMX,IAMNMX)
      CALL MINMAX (AA,    NB,AMNMX,IAMNMX)
      IF (JPOINT(1) .EQ. 0) JPOINT(1) = JDUM
      GO TO 300
C
  240 JPOINT(1) = JDUM
      JPOINT(2) = JDUM
      CALL MINMAX (AA,NB,AMNMX,IAMNMX)
      GO TO 300
C
C----MAIN LOOP FOR READING X, YS----
C
  300 READ (JF,5300,END=500) JDUM,XNEW,(AA(II),II=1,NB)
 5300 FORMAT (I10, 51(1PE14.5) )
      IF (JDUM .EQ. -999) GO TO 500
        IPOINT = IPOINT + 1
        JPOINT(2) = JDUM
        CALL MINMAX (AA,NB,AMNMX,IAMNMX)
C
        IF (IND .NE. 0) GO TO 310
          XAXMAX = XNEW
          OMLAST(J) = XAXMAX
          GO TO 300
  310   IF (XNEW .LT. XAXMAX) GO TO 300
          GO TO 500
C
  500 IF (IND .EQ. 1) GO TO 505
  502   WRITE (6,6502)
```

```
  503   READ (5,5503,ERR=503) IANS
        IF (IANS .EQ. 1HN) RETURN
C--  504   WRITE (6,6050) JF
        JFM50 = JF -50
  504   WRITE (6, 6050) JFM50, FNAME(JFM50)
  505 CONTINUE
C
  506 WRITE (6,6506) (IDUMMY(II),II=1,15)
C
  508 WRITE (6,6508) NB
C
  510 WRITE (6,6510) XAXMIN, XAXMAX, (JPOINT(I),I=1,2)
C
  520 WRITE (6,6520) (AMNMX(I),IAMNMX(I),I=1,2)
C
  530 WRITE (6,6530) IPOINT
C
        RETURN
C
  600 WRITE (6,6600)
      RETURN
C
  650 WRITE (6,6650)
      RETURN
C
 5035 FORMAT (A1)
C
 5503 FORMAT (A1)
C
 6030 FORMAT (/2X, 'DISPLAY OUTPUT SUMMARY ? (Y/N).:  '$)
C
 6050 FORMAT (/2X, 'OUTPUT SUMMARY FOR FILE ', I2, ' :',
     1         /2X, '------------------------',   '----',
     2         /2X, 'FILE NAME :  ', A )
C
 6502 FORMAT (/2X, 'DISPLAY OUTPUT SUMMARY ? (Y/N).:  '$)
C
 6506 FORMAT (/5X,    'RUN TITLE :  ', 15A4)
C
 6508 FORMAT (/5X,    'NUMBER OF Y COLUMNS.........:  ', I5)
C
 6510 FORMAT (/5X,    'X VALUE RANGE :',
     1        /5X,    'X MIN...................:  ', 1PE12.4,
     2        /5X,    'X MAX...................:  ', 1PE12.4,
     3       //5X,    'POINTS RANGED FROM ', I7, ' TO ', I7 )
C
 6520 FORMAT (/5X,    'MINIMUM Y VALUES ON RANGE :',
     1        /5X,    'Y MIN...................:  ', 1PE12.4,
     3        /5X,    'IN COLUMN...............:  ', I12,
     4       //5X,    'MAXIMUM Y VALUES ON RANGE :',
     5        /5X,    'Y MAX...................:  ', 1PE12.4,
     7        /5X,    'IN COLUMN...............:  ', I12    )
C
 6530 FORMAT (/5X,    'TOTAL POINTS IN RANGE..:  ', I12 )
C
 6600 FORMAT (/2X, 'ERROR IN FILE SEARCH - ',
     1             'NO VALID POINTS FOUND.'  )
C
 6650 FORMAT (/2X, 'NO POINTS FOUND IN SPECIFIED RANGE.')
C
      END
```

```
C
      SUBROUTINE SETTRN
C
C---
C
      COMMON /INP1/ OMINIT(20), OMLAST(20),
     1              ICPS, ITERM
      COMMON /INP2/ IRUNTL(20,15), ITITLE(50), IDUMMY(15)
C
      COMMON /INP3/ MAXPTS(20), MAXBLD(20), NBL(20)
      COMMON /INP4/ NFILES, NCURVE(20), ITOTCV, NBLADE(20,50)
      COMMON /INP5/ FNAME(20)
C
      COMMON /LABS/ IXLAB(30), IYLAB(30)
      COMMON /CHAR/ ICHAXS, ICHTIT, JXSPAC, KYSPAC, LTSPAC
C
      COMMON /AXS / XAXMIN, XAXMAX, YAXMIN, YAXMAX
      COMMON /TRAN/ IXTRAN, IYTRAN
C
      COMMON /SCRN/ IXSCMN, IXSCMX, IYSCMN, IYSCMX,
     1              IMINTX, IMINTY, IMAJTX, IMAJTY
C
      COMMON /LEGS/ LFLAG,IDESC(1000),JDESC(1000,20),MXDESC,JCV(10)
      CHARACTER * 100 FNAME
C
C---
C
  100 WRITE (6,6100)
  110 READ (5,5110,ERR=100) IANS
 5110 FORMAT (A1)
C
      IF (IANS .EQ. 1HY) RETURN
C
  120 WRITE (6,6120)
      READ (5,*,ERR=120) IXTRAN, IYTRAN
C
      IF ( (IXTRAN .NE. 1) .AND.
     1     (IXTRAN .NE. 2) ) GO TO 120
      IF ( (IYTRAN .NE. 1) .AND.
     1     (IYTRAN .NE. 2) ) GO TO 120
C
      RETURN
C
C 6100 FORMAT (/2X, 'LINEAR CARTESIAN GRID OK? (Y/N):  '$)
 6100 FORMAT (/2X, 'SEMI-LOGARITHMIC GRID OK? (Y/N):  '$)
C
 6120 FORMAT (/2X, 'AXIS TRANSFORM INDICES :',
     1        /2X, '     = 1 -- LINEAR      ',
     2        /2X, '     = 2 -- LOG         ',
     3        /2X, 'ENTER IXTRAN, IYTRAN...........:  '$)
C
      END
      SUBROUTINE NCHARX (IA,ICHAR,NN)
C
      DIMENSION IA(NN)
      DATA IBLNK / 1H /
C
      DO 110  I=1,NN
        IF (IA(NN-I+1) .NE. IBLNK) GO TO 120
  110 CONTINUE
C
```

```
   120 ICHAR = NN-I+1
C
       RETURN
       END
C
       SUBROUTINE SETTEK
C
       COMMON /INP1/ OMINIT(20), OMLAST(20),
     1               ICPS, ITERM
       COMMON /INP2/ IRUNTL(20,15), ITITLE(50), IDUMMY(15)
C
       COMMON /INP3/ MAXPTS(20), MAXBLD(20), NBL(20)
       COMMON /INP4/ NFILES, NCURVE(20), ITOTCV, NBLADE(20,50)
       COMMON /INP5/ FNAME(20)
C
       COMMON /LABS/ IXLAB(30), IYLAB(30)
       COMMON /CHAR/ ICHAXS, ICHTIT, JXSPAC, KYSPAC, LTSPAC
C
       COMMON /AXS / XAXMIN, XAXMAX, YAXMIN, YAXMAX
       COMMON /TRAN/ IXTRAN, IYTRAN
C
       COMMON /SCRN/ IXSCMN, IXSCMX, IYSCMN, IYSCMX,
     1               IMINTX, IMINTY, IMAJTX, IMAJTY
C
       COMMON /LEGS/ LFLAG,IDESC(1000),JDESC(1000,20),MXDESC,JCV(10)
       CHARACTER * 100 FNAME
   100 WRITE (6,6100)
       READ (5,*,ERR=100) ICPS
       IF ( (ICPS .NE.  30) .AND.
     1      (ICPS .NE.  60) .AND.
     2      (ICPS .NE. 120) .AND.
     3      (ICPS .NE. 480) .AND.
     4      (ICPS .NE. 960) ) GO TO 100
C
   200 WRITE (6,6200)
       READ (5,*,ERR=200) IDUM
       ITERM = 0
C
       IF ( (IDUM .EQ. 4014) .OR.
     1      (IDUM .EQ. 4015) ) ITERM = 3
       IF ( (IDUM .EQ. 4010) .OR.
     1      (IDUM .EQ. 4012) .OR.
     2      (IDUM .EQ. 4051) .OR.
     3      (IDUM .EQ. 4052) ) ITERM = 1
C
       IF ( (ITERM .NE. 1) .AND.
     1      (ITERM .NE. 3) ) GO TO 200
C
       RETURN
C
  6100 FORMAT (/2X, 'ENTER CHAR PER SECOND..........:  '$)
  6200 FORMAT (/2X, 'ENTER TEK TERMINAL TYPE........:  '$)
C
       END
       SUBROUTINE INPLOT
C
       COMMON /INP1/ OMINIT(20), OMLAST(20),
     1               ICPS, ITERM
       COMMON /INP2/ IRUNTL(20,15), ITITLE(50), IDUMMY(15)
C
       COMMON /INP3/ MAXPTS(20), MAXBLD(20), NBL(20)
```

```
      COMMON /INP4/ NFILES, NCURVE(20), ITOTCV, NBLADE(20,50)
      COMMON /INP5/ FNAME(20)
      COMMON /INP6/ XMNINP, XMXINP, YMNINP, YMXINP
C
      COMMON /LABS/ IXLAB(30), IYLAB(30)
      COMMON /CHAR/ ICHAXS, ICHTIT, JXSPAC, KYSPAC, LTSPAC
C
      COMMON /AXS / XAXMIN, XAXMAX, YAXMIN, YAXMAX
      COMMON /TRAN/ IXTRAN, IYTRAN
C
      COMMON /SCRN/ IXSCMN, IXSCMX, IYSCMN, IYSCMX,
     1              IMINTX, IMINTY, IMAJTX, IMAJTY
C
      COMMON /LEGS/ LFLAG, IDESC(1000), JDESC(1000,20), MXDESC, JCV(10)
      CHARACTER * 100 FNAME
C
C...SET SOME OF THE PLOTTING PARAMETERS INTERNALLY.
C
      CALL SETJCV
C
      IXTRAN = 1
      IYTRAN = 2
C
      CALL SETTRN
C
      IMINTX = 2
      IMINTY = 2
C
      IMAJTX = 5
      IMAJTY = 5
C
      ICHAXS = 1
      ICHTIT = 1
      IF (ITERM .EQ. 3) ICHAXS = 2
      IF (ITERM .EQ. 3) ICHTIT = 1
C
      JXSPAC = 4
      KYSPAC = 7
      IF (ITERM .EQ. 3) KYSPAC=6
      LTSPAC = 1
C
      IXSCMN = 450
      IXSCMX = 3800
      IYSCMN = 350
      IYSCMX = 3000
C
      DO 40  J=1,NFILES
        JF = J+50
   40 WRITE (6,6040) J, OMINIT(J), OMLAST(J)
C
   50 WRITE (6,6050)
      READ (5,*,ERR=50) XAXMIN, XAXMAX
C
      DO 60  J=1,NFILES
        CALL OUTSUM(J,1)
   60 CONTINUE
C
C
   82 WRITE (6,6082)
      READ (5,*,ERR=82) YAXMIN, YAXMAX
C
```

```
      XMNINP = XAXMIN
      XMXINP = XAXMAX
      YMNINP = YAXMIN
      YMXINP = YAXMAX
C
      ITOTCV = 0
C
      DO 95  J=1,NFILES
        JF = J+50
   84   WRITE (6,6084) J
        READ (5,*,ERR=84) NCURVE(J)
        IF (NCURVE(J) .GT.  50) NCURVE(J)=50
        IF (NCURVE(J) .LT.   0) GO TO 84
C
        NCJ = NCURVE(J)
        DO 90  K=1,NCJ
   86     WRITE (6,6086) K
          READ (5,*,ERR=86) NBLADE(J,K)
            IF ((NBLADE(J,K) .GT.  0) .AND.
     1           (NBLADE(J,K) .LE.  NBL(J))) GO TO 88
C
   87       WRITE (6, 6087)
            GO TO 86
   88     CONTINUE
          ITOTCV = ITOTCV + 1
   90   CONTINUE
C
   95 CONTINUE
C
      CALL SETLEG
C
  100 WRITE (6,6100)
      READ (5,5100,ERR=100) (IXLAB(II), II=1,30)
C
  110 WRITE (6,6110)
      READ (5,5110,ERR=110) (IYLAB(II), II=1,30)
C
  140 WRITE (6,6140)
      READ (5,5140,ERR=140) (ITITLE(II), II=1,50)
C
  150 CONTINUE
C
C...INITIALIZE PLOTS.
C
  200 CALL INITT(ICPS)
      CALL TERM(ITERM,4096)
C
      RETURN
C
 6040 FORMAT (/2X, 'FOR FILE ', I2, ' :',
     1           /2X, 'MIN X FOR RUN :', 1PE14.5,
     2           /2X, 'MAX X FOR RUN :', 1PE14.5 )
C
 6050 FORMAT (/2X, 'ENTER X-MIN, X-MAX FOR PLOT....:  '$)
C
 6060 FORMAT (/2X, 'FOR TAPE', I2, ' ON ',
     1           /2X, 'SPECIFIED FREQ. RANGE :',
     2          /2X, 'MAX AMP WAS ON BLADE :  ', I2)
C
 6070 FORMAT ( /2X, 'MIN AMP FOR BLADE WAS :', 1PE14.5,
     1           /2X, 'MAX AMP FOR BLADE WAS :', 1PE14.5 )
```

```
C
 6082 FORMAT (/2X, 'ENTER Y-MIN, Y-MAX FOR PLOT....: '$)
C
 6084 FORMAT (/2X, 'ENTER NUMBER OF CURVES',
     1     /2X, 'FOR FILE ', I2, ' ...................: '$)
C
 6086 FORMAT ( /2X, 'FOR THIS FILE, ',
     1          /2X, 'ENTER COLUMN NO. FOR CURVE ',
     2               I2, ' .: ' $)
C
 6087 FORMAT ( //5X, 'BAD INPUT COLUMN NUMBER. ', //)
C
 6100 FORMAT ( /2X, 'ENTER X-AXIS LABEL (MAX 30 CH) : '$)
 5100 FORMAT (30A1)
C
 6110 FORMAT ( /2X, 'ENTER Y-AXIS LABEL (MAX 30 CH) : '$)
 5110 FORMAT (30A1)
C
 6140 FORMAT ( /2X, 'ENTER PLOT TITLE...............: '$)
 5140 FORMAT (50A1)
C
      END
C
      SUBROUTINE TRIM
C
C
      COMMON /INP1/ OMINIT(20), OMLAST(20),
     1              ICPS, ITERM
      COMMON /INP2/ IRUNTL(20,15), ITITLE(50), IDUMMY(15)
C
      COMMON /INP3/ MAXPTS(20), MAXBLD(20), NBL(20)
      COMMON /INP4/ NFILES, NCURVE(20), ITOTCV, NBLADE(20,50)
      COMMON /INP5/ FNAME(20)
C
      COMMON /LABS/ IXLAB(30), IYLAB(30)
      COMMON /CHAR/ ICHAXS, ICHTIT, JXSPAC, KYSPAC, LTSPAC
C
      COMMON /AXS / XAXMIN, XAXMAX, YAXMIN, YAXMAX
      COMMON /TRAN/ IXTRAN, IYTRAN
C
      COMMON /SCRN/ IXSCMN, IXSCMX, IYSCMN, IYSCMX,
     1              IMINTX, IMINTY, IMAJTX, IMAJTY
C
      COMMON /LEGS/ LFLAG,IDESC(1000),JDESC(1000,20),MXDESC,JCV(10)
      CHARACTER * 100 FNAME
C
      DIMENSION JCHPTS(4,2)
C
C...JCHPTS ARE THE RELATIVE CHARACTER SCREEN SIZES
C   AS APPEAR ON THE PLOT. REF. PLOT10 MANUAL.
C
      DATA JCHPTS /56, 51, 34, 31,
     +             88, 83, 53, 48/
C
C
      IF (ITERM .EQ. 3) CALL CHRSIZ(ICHAXS)
C
C
C...PROCESS X-AXIS LABEL.
C
      CALL NCHARX (IXLAB,IXCHAR,30)
```

```
      XMID = (XAXMAX-XAXMIN)/2.   +  XAXMIN
      IF (IXTRAN .EQ. 2) XMID = SQRT(XAXMAX*XAXMIN)
C
      CALL MOVEA (XMID,YAXMIN)
C
      IXX = -(IXCHAR * JCHPTS(ICHAXS,1) )/2
C
      IYY = -(JXSPAC * JCHPTS(ICHAXS,2) )
C
      CALL MOVREL (IXX,IYY)
      CALL ANMODE
      DO 130  I=1,IXCHAR
        CALL AOUTST(1,IXLAB(I))
  130 CONTINUE
C
C...PROCESS Y-AXIS LABEL.
C
      CALL NCHARX (IYLAB,IYCHAR,30)
      YMID = (YAXMAX-YAXMIN)/2.   +  YAXMIN
      IF (IYTRAN .EQ. 2) YMID = SQRT (YAXMAX*YAXMIN)
      CALL MOVEA (XAXMIN,YMID)
C
      IXX = -(KYSPAC * JCHPTS(ICHAXS,1) )
C
      IYY = (IYCHAR * JCHPTS(ICHAXS,2) )/2
C
      CALL MOVREL (IXX,IYY)
      JXX = -JCHPTS(ICHAXS,1)
      JYY = -JCHPTS(ICHAXS,2)
C
      CALL ANMODE
      DO 140  I=1,IYCHAR
        CALL AOUTST(1,IYLAB(I))
        CALL MOVREL(JXX,JYY)
  140 CONTINUE
C
C...PROCESS TITLE.
C
C     CALL CHRSIZ(ICHTIT)
      IF (ITERM .EQ. 3) CALL CHRSIZ(ICHTIT)
      CALL NCHARX (ITITLE,ITCHAR,50)
C
C...TITLE CENTERED AT PLOT VERT.  MIDLINE.
C
      CALL MOVEA (XMID,YAXMAX)
      IF (LTSPAC .LT. 0) CALL MOVEA (XMID,YAXMIN)
C
      IXX = -(ITCHAR * JCHPTS(ICHTIT,1) )/2
C
      IYY = LTSPAC * JCHPTS(ICHTIT,2)
C
      CALL MOVREL (IXX,IYY)
      CALL ANMODE
      DO 150  I=1,ITCHAR
        CALL AOUTST(1,ITITLE(I))
  150 CONTINUE
C
C     CALL ANMODE
C     CALL CHRSIZ(4)
      IF (ITERM .EQ. 3) CALL CHRSIZ(4)
C
```

```
      RETURN
      END
C
      SUBROUTINE FNPLOT
C
C......   CLEAN UP PLOT10 DETAILS.
C
C     CALL CHRSIZ(4)
      IF (ITERM .EQ. 3) CALL CHRSIZ(4)
C
C......   PAUSE FOR POSIBLE HARD-COPY.
C
C     CALL ANMODE
C      CALL VCURSR (IDUM, XDUM, YDUM)
      CALL FINITT(0, 2700)
      RETURN
      END
C
C
      SUBROUTINE LEGDRW
C
      COMMON /INP1/ OMINIT(20), OMLAST(20),
     1              ICPS, ITERM
      COMMON /INP2/ IRUNTL(20,15), ITITLE(50), IDUMMY(15)
C
      COMMON /INP3/ MAXPTS(20), MAXBLD(20), NBL(20)
      COMMON /INP4/ NFILES, NCURVE(20), ITOTCV, NBLADE(20,50)
      COMMON /INP5/ FNAME(20)
C
      COMMON /LABS/ IXLAB(30), IYLAB(30)
      COMMON /CHAR/ ICHAXS, ICHTIT, JXSPAC, KYSPAC, LTSPAC
C
      COMMON /AXS / XAXMIN, XAXMAX, YAXMIN, YAXMAX
      COMMON /TRAN/ IXTRAN, IYTRAN
C
      COMMON /SCRN/ IXSCMN, IXSCMX, IYSCMN, IYSCMX,
     1              IMINTX, IMINTY, IMAJTX, IMAJTY
C
      COMMON /LEGS/ LFLAG, IDESC(1000), JDESC(1000,20), MXDESC, JCV(10)
      CHARACTER * 100 FNAME
C
      DIMENSION JCHPTS(4,2)
      DIMENSION LEGSTR(6), LINSTR(6)
C
      DATA JCHPTS /56, 51, 34, 31,
     1             88, 83, 53, 48 /
C
      DATA LEGSTR /1HL, 1HE, 1HG, 1HE, 1HN, 1HD/
      DATA LINSTR /1H-, 1H-, 1H-, 1H-, 1H-, 1H-/
      DATA COLSTR /1H:/
C
C
      IF (LFLAG .EQ. 0) RETURN
C
C......   PLOT LEGEND.
C
C
      ICHR = 1
      IF (ITERM .EQ. 3) ICHR = 3
      IBEG = ( (MXDESC+9)/2 ) - 3
      ILONG = (MXDESC+9) * JCHPTS(ICHR,1)
```

```
C
      FAC1 = 2.5 + (1.1*FLOAT(ITOTCV))
      FAC2 = FAC1 * FLOAT(JCHPTS(ICHR,2))
C
      IWIDE = FAC2
C
C     CALL CHRSIZ(ICHR)
      IF (ITERM .EQ. 3) CALL CHRSIZ(ICHR)
C
      CALL VCURSR (L,XL,YL)
      CALL MOVEA (XL,YL)
C
      IXX = IBEG * JCHPTS(ICHR,1)
      IYY = -1   * JCHPTS(ICHR,2)
C
      CALL MOVREL (IXX,IYY)
      CALL ANMODE
      DO 10  I=1,6
        CALL AOUTST (1,LEGSTR(I))
   10 CONTINUE
C
      IXX = -6 * JCHPTS(ICHR,1)
      IYY = -1 * JCHPTS(ICHR,2)
C
      CALL MOVREL (IXX,IYY)
      CALL ANMODE
      DO 15  I=1,6
        CALL AOUTST(1,LINSTR(I))
   15 CONTINUE
C
      HALF = 0.3 * FLOAT(JCHPTS(ICHR,2))
      IHALF = HALF
C
      IXX = JCHPTS(ICHR,1)
      ILINE = 4 * JCHPTS(ICHR,1)
C
      DO 20  II=1,ITOTCV
        IJCV = JCV(II)
        CALL MOVEA (XL,YL)
        FCTR = 2. + (1.1*FLOAT(II))
C
        DOWN = -(FCTR * FLOAT(JCHPTS(ICHR,2)))
        IDOWN = DOWN
C
        IUP = IDOWN + IHALF
C
        CALL MOVREL (IXX,IUP)
        CALL DSHREL (ILINE,0,IJCV)
C
        CALL MOVREL (IXX,-IHALF)
        CALL ANMODE
        CALL AOUTST (1,COLSTR)
C
        CALL MOVREL (IXX,0)
        IDKD = IDESC(II)
        CALL ANMODE
        DO 18  JJ=1,IDKD
          KDUMMY = JDESC(II,JJ)
          CALL AOUTST(1,KDUMMY)
   18   CONTINUE
C
```

```
   20 CONTINUE
C
      CALL MOVEA (XL,YL)
C
      CALL DRWREL (ILONG,0)
      CALL DRWREL (0,-IWIDE)
      CALL DRWREL (-ILONG,0)
      CALL DRWREL (0,IWIDE)
C
      RETURN
      END
C
      SUBROUTINE PLODER
C
C---
C
      COMMON /INP1/ OMINIT(20), OMLAST(20),
     1              ICPS, ITERM
      COMMON /INP2/ IRUNTL(20,15), ITITLE(50), IDUMMY(15)
C
      COMMON /INP3/ MAXPTS(20), MAXBLD(20), NBL(20)
      COMMON /INP4/ NFILES, NCURVE(20), ITOTCV, NBLADE(20,50)
      COMMON /INP5/ FNAME(20)
      COMMON /INP6/ XMNINP, XMXINP, YMNINP, YMXINP
C
      COMMON /LABS/ IXLAB(30), IYLAB(30)
      COMMON /CHAR/ ICHAXS, ICHTIT, JXSPAC, KYSPAC, LTSPAC
C
      COMMON /AXS / XAXMIN, XAXMAX, YAXMIN, YAXMAX
      COMMON /TRAN/ IXTRAN, IYTRAN
C
      COMMON /SCRN/ IXSCMN, IXSCMX, IYSCMN, IYSCMX,
     1              IMINTX, IMINTY, IMAJTX, IMAJTY
C
      COMMON /LEGS/ LFLAG,IDESC(1000),JDESC(1000,20),MXDESC,JCV(10)
      CHARACTER * 100 FNAME
C---
C
      DIMENSION XDUM(2), YDUM(2)
      DIMENSION AA(50)
C
C-----DRAW AXES, FRAME, AXIS LABELS-----
C
      CALL BINITT
C
      IF (ITERM .EQ. 3) CALL CHRSIZ(3)
C
      CALL XMFRM(IMINTX)
      CALL YMFRM(IMINTY)
C
      CALL XFRM(IMAJTX)
      CALL YFRM(IMAJTY)
C
      CALL SLIMX(IXSCMN,IXSCMX)
      CALL SLIMY(IYSCMN,IYSCMX)
C
      CALL XTYPE(IXTRAN)
      CALL YTYPE(IYTRAN)
C
      CALL DLIMX(XMNINP,XMXINP)
      CALL DLIMY(YMNINP,YMXINP)
```

```
C
      XDUM(1) = 1.
      XDUM(2) = XMNINP
      YDUM(1) = 1.
      YDUM(2) = YMNINP
C
      IF (ITERM .EQ. 3) CALL CHRSIZ(3)
C
      CALL CHECK (XDUM,YDUM)
      CALL DSPLAY (XDUM,YDUM)
      CALL FRAME
C
C----RESET AX MINS, MAXS----
C
      IX1 = IBASEX(11)
      IX2 = IBASEX(12)
C
      XAXMIN = COMGET(IX1)
      XAXMAX = COMGET(IX2)
C
      IY1 = IBASEY(11)
      IY2 = IBASEY(12)
C
      YAXMIN = COMGET(IY1)
      YAXMAX = COMGET(IY2)
C----PLOT CURVES----
C
      ICV = 0
C
      DO 500  J=1,NFILES
        JF = J + 50
        NCJ = NCURVE(J)
        DO 400  K=1,NCJ
          REWIND JF
          ICOL = NBLADE(J,K)
          ICV = ICV + 1
          IF (ICV .GT. 10) ICV=1
          ITYP = JCV(ICV)
C
  310     READ (JF,5310) (IDUMMY(II),II=1,15)
  320     READ (JF,5320) NBDUM
          NB = NBL(J)
          INEW = 1
C
C----SEARCH FOR FIRST X-VAL----
C
  330     READ (JF,5330,END=400) JDUMMY,XNEW,(AA(II),II=1,NB)
          IF (JDUMMY .EQ. -999) GO TO 400
          YNEW = AA(ICOL)
          IF (XNEW .LT. XAXMIN) THEN
            XOLD = XNEW
            YOLD = YNEW
            INEW = 0
            GO TO 330
          ELSE
            IF ( (XNEW .EQ. XAXMIN) .OR.
     1           (INEW  .EQ. 1)  )  THEN
              CALL MOVEA(XNEW,YNEW)
            ELSE
              CALL MOVEA(XOLD,YOLD)
            END IF
```

```
            END IF
C
C----PLOT UNTIL XNEW .GT. XAXMAX OR----
C     JDUMMY .EQ. -999
C
  350       CALL DASHA (XNEW,YNEW,ITYP)
C
  360       READ (JF,5360,END=400) JDUMMY,XNEW,(AA(II),II=1,NB)
            IF (JDUMMY .EQ. -999) GO TO 400
            YNEW = AA(ICOL)
            IF (XNEW .GE. XAXMAX) THEN
              CALL DASHA (XNEW,YNEW,ITYP)
              GO TO 400
            ELSE
              GO TO 350
            END IF
C
  400     CONTINUE
  500 CONTINUE
C
      RETURN
 5310 FORMAT (15A4)
 5320 FORMAT (I10)
 5330 FORMAT (I10, 51(1PE14.5) )
 5360 FORMAT (I10, 51(1PE14.5) )
C
      END
C
C
      SUBROUTINE FINITT(IX,IY)
C
      CALL MOVABS(IX,IY)
      CALL ANMODE
      RETURN
      END
C
      SUBROUTINE DRCHEK (IANS)
C
      COMMON /INP1/ OMINIT(20), OMLAST(20),
     1              ICPS, ITERM
      COMMON /INP2/ IRUNTL(20,15), ITITLE(50), IDUMMY(15)
C
      COMMON /INP3/ MAXPTS(20), MAXBLD(20), NBL(20)
      COMMON /INP4/ NFILES, NCURVE(20), ITOTCV, NBLADE(20,50)
      COMMON /INP5/ FNAME(20)
C
      COMMON /LABS/ IXLAB(30), IYLAB(30)
      COMMON /CHAR/ ICHAXS, ICHTIT, JXSPAC, KYSPAC, LTSPAC
C
      COMMON /AXS / XAXMIN, XAXMAX, YAXMIN, YAXMAX
      COMMON /TRAN/ IXTRAN, IYTRAN
C
      COMMON /SCRN/ IXSCMN, IXSCMX, IYSCMN, IYSCMX,
     1              IMINTX, IMINTY, IMAJTX, IMAJTY
C
      COMMON /LEGS/ LFLAG,IDESC(1000),JDESC(1000,20),MXDESC,JCV(10)
      CHARACTER * 100 FNAME
C
C-----SUBROUTINE TO CHECK FOR REDRAW WITH
C     EXACTLY THE SAME DATA. ESSENTIALLY THIS
C     TAKES CARE OF USER IF HE BLOWS THE
```

```
C     LEGEND POSITIONING.   TH 11/18/82
C
      CALL VCURSR (IDUM, XDUM, YDUM)
      CALL MOVABS (0, 2900)
      CALL ANMODE
  100 WRITE (6, 6100)
  110 READ (5, 5110, ERR=100) IANS
      IF (IANS .EQ. 1HY) CALL NEWPAG
      RETURN
C
 6100 FORMAT (/2X, 'SAME PLOT AGAIN ? (Y/N) .......:  '$)
 5110 FORMAT (A1)
C
      END
```

COMMAND FILE GENERATOR PROC.COM

```
$!
$!  --COMMAND FILE GENERATOR--
$!
$ HEADER:
$ HEAD1:
$  USER := 'F$DIRECTORY()'
$  LEN1 = 'F$LOCATE(".",USER)
$  LEN2 = 'F$LENGTH(USER)'
$  IF (LEN1 .EQ. LEN2) THEN LEN1=LEN1-1
$  MAIN := 'F$EXTRACT (0, LEN1, USER)'"]"
$  TODAY := 'F$TIME()'
$  LAST_SPOT = 'F$LOCATE(":",TODAY)' +3
$  MID_SPOT = LAST_SPOT-2
$  TODAY := 'F$EXTRACT(0,LAST_SPOT,TODAY)'
$  CLOCK_TIME := 'F$EXTRACT(MID_SPOT,5,TODAY)'
$!
$ HEAD2:
$  WRITE SYS$OUTPUT "  "
$  WRITE SYS$OUTPUT "  "
$  WRITE SYS$OUTPUT "****************************",-
      "*********************************"
$  WRITE SYS$OUTPUT "*                            ",-
      "                                *"
$  WRITE SYS$OUTPUT "*                 BLADE BATCH",-
      " INPUT PROCEDURE                *"
$  WRITE SYS$OUTPUT "*                 -----------",-
      "----------------                *"
$  WRITE SYS$OUTPUT "           TERMINAL SESSION :",-
      "  ", TODAY
$  WRITE SYS$OUTPUT "*           -----------------",-
      "-----------------------"
$  WRITE SYS$OUTPUT "*                            ",-
      "                                *"
$  WRITE SYS$OUTPUT "****************************",-
      "*********************************"
$  WRITE SYS$OUTPUT "  "
$!
$ FILE1:
$  ON CONTROL_Y THEN GOTO TERMINUS
$  WRITE SYS$OUTPUT "  "
$  INQUIRE NEWCHK -
   "  CREATE NEW DATA FILE? (Y/N)......."
$  WRITE SYS$OUTPUT "  "
$  IF (NEWCHK .EQS. "Y") THEN GOTO FILE5
$!
$ FILE2:
$  INQUIRE FNAME -
   "  ENTER OLD DATA FILE NAME.TYPE....."
$  WRITE SYS$OUTPUT "  "
$  ON WARNING THEN GOTO FILE3
$  DIRECTORY/OUTPUT=SCRATCH.TXT 'FNAME'
$  GOTO FILE4
$!
$ FILE3:
$  WRITE SYS$OUTPUT "  ** WARNING -",-
      " DATA FILE NOT FOUND IN DIRECTORY **"
$  WRITE SYS$OUTPUT "  "
$  GOTO FILE4
$!
$ FILE4:
$  SET NOON
$  DELETE SCRATCH.TXT;0
```

```
$  GOTO FILE6
$!
$ FILE5:
$  WRITE SYS$OUTPUT "  "
$!
$  INQUIRE FNAME -
   "  ENTER NEW DATA FILE NAME.TYPE..... "
$  WRITE SYS$OUTPUT "  "
$  ASSIGN/USER_MODE SYS$COMMAND FOR005
$  ASSIGN/USER_MODE 'FNAME'   DFF
$  RUN DATAFORM
$  WRITE SYS$OUTPUT "  "
$!  RENAME DFF.DAT 'FNAME'
$  GOTO FILE6
$!
$ FILE6:
$  GOTO WRITE1
$!
$ WRITE1:
$  WRITE SYS$OUTPUT "   WRITING COMMAND FILE... "
$  WRITE SYS$OUTPUT "  "
$  OPEN/WRITE BB JCL.COM
$!
$ WRITE2:
$  WRITE BB "$ SET VERIFY"
$  WRITE BB "$!"
$  WRITE BB "$!   BLADE BATCH INPUT PROCEDURE"
$  WRITE BB "$!      USER : ", USER
$  WRITE BB "$!   SESSION : ", TODAY
$  WRITE BB "$!"
$  WRITE BB "$ ON ERROR THEN GOTO TERMINUS"
$  WRITE BB "$ SET DEF ", USER
$!
$ WRITE3:
$  WRITE BB "$   ASSIGN/USER_MODE ", FNAME,-
      " FOR009"
$  WRITE BB "$   ASSIGN          ",-
           "JCL.LOG SYS$PRINT"
$  WRITE BB "$!"
$  WRITE BB "$   RUN BLADE"
$  WRITE BB "$!"
$!
$ WRITE4:
$  WRITE BB "$!"
$  WRITE BB "$ TERMINUS:"
$  WRITE BB "$  SET NOON"
$  WRITE BB "$  DELETE FOR002.DAT;0"
$  WRITE BB "$  SET ON"
$  WRITE BB "$  SET NOVERIFY"
$  WRITE BB "$  EXIT"
$!
$ WRITE5:
$  CLOSE BB
$  DIR/OUTPUT=SCRATCH.TXT/VERSION=1 JCL.COM
$  OPEN/READ SCR SCRATCH.TXT
$  READ SCR ABC
$  READ SCR ABC
$  READ SCR ABC
$  READ SCR ABC
$  CLOSE SCR
$  SET NOON
```

```
$  DELETE SCRATCH.TXT;0
$  SPOT1 = 'F$LOCATE(";",ABC)' + 4
$  ABC := 'F$EXTRACT(0,SPOT1,ABC)'
$  WRITE SYS$OUTPUT "    COMMAND FILE WRITTEN",-
      " TO : ", ABC
$  WRITE SYS$OUTPUT " "
$  INQUIRE DISBAT -
   "  DISPLAY PROCEDURE HERE? (Y/N)....."
$  WRITE SYS$OUTPUT " "
$  IF DISBAT .NES. "Y" THEN GOTO TERMINUS
$!
$ WRITE6:
$  TYPE JCL.COM
$  WRITE SYS$OUTPUT " "
$  GOTO TERMINUS
$!
$ TERMINUS:
$  SET ON
$  ON CONTROL_Y THEN EXIT
$  WRITE SYS$OUTPUT "    TERMINAL SESSION ENDED."
$  WRITE SYS$OUTPUT " "
$ EXIT
```

REFERENCES

1. A Muszynska, A., D. I. G. Jones, T. Lagnese, L. Whitford, "On Nonlinear Response of Multiple Blade Systems," Shock and Vibration Bulletin, 51, pt. 3, May 1981, pp. 89-110.

2. R. J. Dominic, "The Analysis by the Lumped Parameter Method of Blade Platform Friction Dampers Used in the High Pressure Fuel Turbopump of the Space Shuttle Main Engine," paper presented at the 54th Shock and Vibration Symposium, Pasadena, CA, October 1983; published in Shock and Vibration Bulletin, 54, May 1984.

3. IMSL Library Reference Manual, IMSL LIB-0009, June 1982.